separately measured[3,5,6] though their evaluation requires a more complicated mathematical treatment, especially when t is longer than the shortest delayed-neutron lifetime.

$$\overline{\alpha t < 1}$$

Equation (1) may be written as follows:

$$y = \frac{V - 1}{y_\infty} = \sum_{n=2}^{n=\infty} (-1)^n \frac{(\alpha t)^{n-1}}{n!} . \qquad (3)$$

We find the third zone the most interesting and have focused our study there. Limiting the expansion to a certain value of n, one introduces a positive or negative error depending upon the parity of n. However it is not worthwhile to choose high values for n, remembering that the following expression

$$\frac{\sum_{i=1}^{F} R_i^2}{F - (n-1)}$$

(where R_i is the residue, F is the number of measurements, $n - 1$ is the number of parameters) must be minimized.

We have obtained good results for $n = 4$.

The method is successful because of its simplicity and ease of experimental application. Some advantages are:

1) With small αt's a prompt-reactor analysis is really performed, for the condition $|\alpha_2| t \ll \alpha t < 1$ conforms to Eq. (3), which ignores the presence of delayed neutrons.

2) Rossi alpha measurements are made in a region where V is varying rapidly with αt; in fact

$$\dot{y}(t = 0) = \alpha/2.$$

3) Treatment of the data is easier than for a larger range of αt, but sufficient to get α and Y_∞ with the required accuracy.

4) The total time of the measurement is really much shorter than with any other method; in this way the results are less affected by drifts in reactor power (especially when the reactor is kept critical). In fact, the condition $\alpha t < 1$ allows a maximum channel width

$$t < \frac{\tau_0}{\beta}$$

Experimental conditions are based on an approximate evaluation of τ_0 and β.

Then the requirement that

$$c = f \epsilon t$$

(where f is the number of fissions per unit time) has a numerical value with statistical meaning must be satisfied.

This value could be in the range 10 to 100 and still gives information on variance parameters.

Remembering that

$$\bar{c} = \frac{1}{N} \Sigma_j c_j \qquad \langle (\Delta \bar{c})^2 \rangle = \frac{D}{N}$$

$$D = \frac{1}{N} \Sigma_j (c_j - \bar{c})^2 \qquad \langle (\Delta D)^2 \rangle = \frac{2(N-1)D^2}{N^2}$$

$$V = \frac{D}{\bar{c}} \qquad \langle (\Delta V)^2 \rangle \approx \frac{2D^2}{N\bar{c}^2} \left(1 + \frac{D}{2\bar{c}^2}\right)$$

are the mean values, the absolute variance, the relative variance and their variances respectively, it may be seen that experimental values of the variance have the required significance when the number gates, N, is really high ($\gtrsim 10^4$) no matter if $\bar{c}$ is statistically small.

This short time-variance method has been applied in measuring the prompt-neutron lifetime in the organic moderated ROSPO reactor at CSN Casaccia, CNEN.

Measurements have been performed at a power level of 20 mW, giving satisfactory $\bar{c}$ values and requiring negligible corrections for counter dead time (≈ 2 μsec).

Counts were taken with gate widths of 1 to 5 msec using as a multiscaler a LABEN 512-channel analyzer controlled by an external pulse generator.

The result thus obtained ($\tau_0 = 35$ μsec) is in good agreement with earlier measurements and calculations.

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Level Separation Corrections to Resonance Integrals

Materials with complicated resonance structures are almost always present in nuclear reactors to such an extent that it is worthwhile to employ some analytic means for calculating resonance escape probability. The treatments are

usually based on the Wigner formula[1] for infinite homogeneous media,

$$p = \exp -(I/\bar{\xi}),$$

where $\bar{\xi}$ is the mean lethargy gain per collision, and I is the resonance integral

$$I \equiv \int_{-\infty}^{\infty} \frac{\Sigma_A(E)}{\Sigma_T(E)} \frac{dE}{E} . \qquad (1)$$

This equation is exact when hydrogen is the sole moderator. It is valid for other media only if all strongly absorbing resonances are very narrow and widely separated. A recent extension[2] allows widely separated resonances to be less narrow, and the procedure developed in this note allows narrow resonances to be less widely separated. Although the method steadily becomes more cumbersome as the resonance spacing decreases, many practical situations require only a first-order correction to the Wigner formula, since it is usually adequate even by itself.

Large level spacings are required by the Wigner formula for nonhydrogeneous moderators because the flux immediately above each resonance is assumed to be of the form $1/E$. However, when the spacings are not very large, a resonance may lie in the midst of Placzek oscillations produced by other resonances at higher energies. Then there arises the problem of determining: 1) the rate at which a resonance in the Placzek oscillations absorbs neutrons, and 2) how this absorption affects the flux below the resonance. The second problem will be treated here simply by considering each resonance as a negative monoenergetic source equal to its absorption rate.

Before the Wigner formula is generalized, however, the derivation of its original form should be made more cogent than the usual descriptions. Let $q(E)$ be the slowing-down density at E and $J(E)dE$ the rate at which all neutrons with energies above E are scattered into the infinitesimal interval dE just below E. From the relation $J(E) = \Sigma_T \phi(E)$, one can eliminate the flux $\phi(E)$ from the equation $dq/dE = \Sigma_A \phi(E)$, divide through by q, and integrate to obtain

$$\frac{q(E_2)}{q(E_1)} = \exp - \int_{E_2}^{E_1} \frac{\Sigma_A(E)}{\Sigma_T(E)} w(E)dE, \qquad (2)$$

where $w(E) \equiv J(E)/q(E)$.

When hydrogen is the only moderator, $w(E)$ always equals $1/E$, for the probability that a neutron with any energy above E will scatter into dE is rigorously dE/E times the probability that it will scatter below E. However, if other moderators are present, the ratio of probabilities depends on the energy of the neutron before the collision, and therefore $w(E)$ depends on the (unknown) flux above E.

In order to avoid calculating this flux, $w(E)$ is to be approximated, but in order to make the results also practical, the expression for this ratio should be made: 1) exact for pure hydrogen, 2) a simple function of E, and 3) accurate for narrow resonances. The first condition suggests that the approximation for $w(E)$ be taken from an exact calculation based on a fictitious absorption cross section, since such a result will always correctly degenerate to $1/E$ when the moderator is made pure hydrogen. The closer the fictitious cross section is to the exact form, the better the approximation will be when other moderators are present. However, it is difficult to propose a realistic form that always satisfies condition 2). A further retreat is called for, and condition 3) specifies the direction: find a simple fictitious absorption cross section that makes $w(E)$ accurate at least for narrow resonances. The natural fictitious absorption cross section to try is that for infinitely narrow resonances, namely zero. When the actual resonances are narrow, such an approximation for $w(E)$ within them should be accurate because both its numerator $J(E)$ and denominator $q(E)$ are: 1) integrals over energy intervals presumably much larger than that in which the flux dips; and 2) changed by the dip in the same direction, although only the hydrogenous contributions are changed proportionately.

From Eq. (2), the approximation for the resonance escape probability across a narrow resonance is, then, simply

$$p = \exp - \int_{-\infty}^{\infty} \frac{\Sigma_A(E)}{\Sigma_T(E)} w^*(E)dE, \qquad (3)$$

where an asterisk denotes that the value is to be for no absorption. Thus,

$$J^*(E) = \Sigma_S \phi^*(E), \qquad (4)$$

where the scattering cross section Σ_S is assumed constant. Furthermore, *if* the flux immediately above the resonance is $1/E$ then

$$q^*(E) = \bar{\xi} \Sigma_S E \phi^*(E), \qquad (5)$$

and Eqs. (2-5) yield the Wigner formula. However, if the flux immediately above the resonance is *not* $1/E$, Eq. (5) must be modified before being combined with Eqs. (3) and (4).

For this more general case, let u_n be the lethargy of the n'th resonance, numbered in order of increasing lethargy. Let $\Sigma_{An}(u)$ be the absorption cross section of this resonance, and p_n, the

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non-absorption probability across it. Finally, let all lethargy-dependent quantities between resonances n-1 and n be subscripted by n. Within resonance n,

$$J^*(u) = \Sigma_S \phi_n(u) \text{ and } q^*(u) = q_n,$$

so by Eqs. (2) and (3)

$$p_n = \exp - \int_{-\infty}^{\infty} \frac{\Sigma_{An}(u)}{\Sigma_T(n)} w_n(u)du,$$

where

$$w_n(u) \equiv \frac{\Sigma_S \phi_n(u)}{q_n} . \quad (6)$$

The rate at which resonance n absorbs neutrons is $(1-p_n)q_n$, which is to be considered the size of the negative monoenergetic source replacing this absorption. Thus,

$$\phi_{n+1}(u) = \phi_n(u) - (1-p_n)q_n G(u-u_n),$$

where $G(u-u_n)$ is the flux at u from a unit monoenergetic source at u_n. Since $q_{n+1} = p_n q_n$ by definition,

$$w_{n+1}(u) = p_n^{-1} w_n(u) - \Sigma_S(1-p_n) G(u-u_n). \quad (7)$$

In principle, now, Eqs. (6) and (7) could be coupled to give values of p_n for successive values of n. The sequence is initiated by using the Wigner formula for some resonance with $1/E$ flux immediately above.

Although the general form of the function $G(u-u_n)$ is much too erratic to handle, usually only its asymptotic form is needed. The resulting corrections to the Wigner formula will thus be those first required as the level spacing is decreased from infinity. The calculation will now be illustrated for hydrogenous mixtures, and only the principal transient will be retained. A rather unique simplification is that this transient is non-oscillatory[3]. However, it would be straightforward to generalize the procedure to include oscillatory additional transients and cover any mixture of isotopes.

In a nonabsorbing medium, the asymptotic flux per unit lethargy from a monoenergetic source at lethargy u_n is

$$G(u-u_n) \simeq \left[\bar{\xi}^{-1} + a \exp - r(u-u_n)\right] \Sigma_S^{-1}, \quad (8)$$

where r is the nonzero real root to the transcendental equation[3,4]

$$1 - r = \frac{\Sigma_{SH}}{\Sigma_S} + \sum_i \frac{\Sigma_{Si}}{\Sigma_S} \left(\frac{1-\alpha_i \exp r\Delta_i}{1-\alpha_i}\right)$$

and

$$a \equiv \frac{1 - r}{1 - \sum_i \frac{\Sigma_{Si}}{\Sigma_S} \left(\frac{\alpha_i \Delta_i \exp r\Delta_i}{1-\alpha_i}\right)} .$$

Here, Σ_{SH} is the scattering cross section of hydrogen, Σ_{Si} and Δ_i are that and the maximum lethargy gain per collision for a nonhydrogenous isotope i, and $\alpha_i \equiv \exp - \Delta_i$.

Now, $w_n(u)$ is always of the form

$$w_n(u) = \bar{\xi}^{-1} - \beta_n \exp - r(u-u_n). \quad (9)$$

Equations (7-9) give

$$\beta_{n+1} = p_n^{-1} \left[\beta_n + (1-p_n)\, a\right] \exp - r(u_{n+1}-u_n). \quad (10)$$

Successive p_n's can be generated by coupling Eq. (10) with the results of Eqs. (6) and (9), namely

$$p_n = p_{nw} \exp \beta_n \int_{-\infty}^{\infty} \frac{\Sigma_{An}(u)}{\Sigma_T(u)} \exp - r(u-u_n)du, \quad (11)$$

where

$$p_{nw} \equiv \exp - \frac{I_n}{\bar{\xi}}, \text{ with } I_n \equiv \int_{-\infty}^{\infty} \frac{\Sigma_{An}(u)}{\Sigma_T(u)} du, \quad (12)$$

is the (known) Wigner, or wide spacing, formula for the resonance escape probability across resonance n. The sequence is started by setting $\beta_n = 0$ for some resonance preceded by a $1/E$ flux. It is easily seen that p_n approaches p_{nw} for wide spacing.

Since the resonances are narrow, in Eq. (1) the $1/E$ factor and a $1/\sqrt{E}$ coefficient in $\Sigma_A(E)$ are usually taken outside the integral sign with E replaced with E_n. It would thus be consistent to replace Eq. (11) with the simple form

$$p_n = p_{nw} \exp \beta_n I_n.$$

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