Short Time-Variance Method for Prompt-Neutron-Lifetime Measurements

Early considerations of fluctuations of the neutron population have appeared in a theoretical treatment by de Hoffmann[1]. They have been extended to include the effects of delayed neutrons by Bennett[2].

If c is the value obtained from counting a neutron detector placed inside the reactor, we have

$$V = \frac{\overline{c^2} - \bar{c}^2}{\bar{c}} = 1 + \sum_i Y_i \left(1 - \frac{1 - e^{\alpha_i t}}{|\alpha_i| t}\right) \quad (i = 1,2,\ldots,7), \tag{1}$$

where

V is the relative variance
t is the gate width
α_i are the poles of the reactor transfer function
Y_i are constants.

From an experimental point of view, Eq. (1) can be simplified and still meaningfully describe the phenomena. Albrecht[3] gathered delayed-neutron contributions and constants (i.e., α_i and Y_i) into two terms so that $(i = 1,2, \ldots . 7)$ becomes $(i = 1,2,3)$ in Eq. (1).

When applied to a single fuel, under fixed reactivity conditions, the parameters appearing in Eq. (1) have the following properties and meaning:

$Y_1 = Y_\infty$, the asymptotic value of correlated 'prompt' variance (2a)

$$0 < Y_1 < Y_2 < Y_3 \tag{2b}$$

$$|\alpha_1| = \frac{1 - K_p}{\tau_0} = \alpha, \text{ the Rossi alpha} \tag{2c}$$

$$|\alpha_3| < |\alpha_2| << |\alpha_1|. \tag{2d}$$

It must also be noted that the smaller the value of $1 - K_p$, the earlier the effect of delayed neutrons manifests itself in Eq. (1). Some numerical calculations by Bennett and by Albrecht confirm the validity of the statement.

These conditions existing, it has been shown that the delayed terms $(i = 2,3)$ do not affect the prompt-term value $(i = 1)$, until the condition $|\alpha_2| t << 1$ is reached, even if Eq. (2b) holds. When t becomes longer, delayed-neutron terms have more numerical weight on variance than do the prompt-neutron terms, because of Eq. (2b).

We have summarized the experimental observations made up to now in Fig. 1, which represents only the prompt-neutron contribution to the variance.

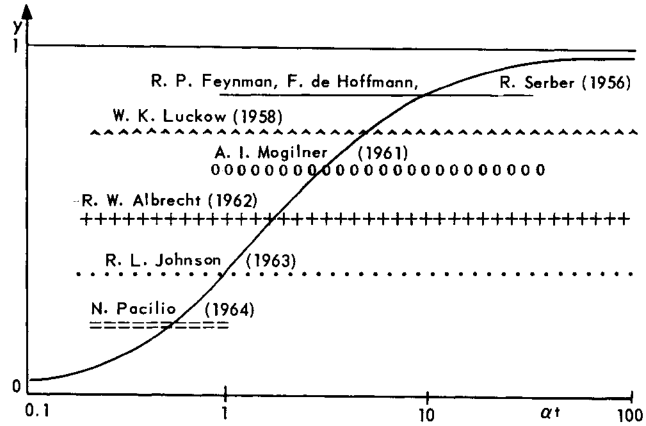

Fig. 1. Experimental values of correlated variance as a function of counting time. (Prompt neutrons only are included.)

We may divide the αt axis into three zones.

$$\alpha t >> 1$$

Equation (1) becomes

$$V = 1 + Y_\infty + \sum_{i=2}^{3} Y_i \left(1 - \frac{1 - e^{\alpha_i t}}{|\alpha_i| t}\right),$$

where $Y = \epsilon D/\beta^2$ for delayed criticality (ϵ is the counter efficiency, β is the total fraction of delayed neutrons and D is calculated from the Diven formula). Feynman *et al.*[4] have considered operating conditions in which the gate width was so short that delayed-neutron effects were avoided and yet long enough to allow the formula to become the simple relation

$$V = 1 + Y_\infty,$$

since

$$\frac{1}{\alpha} << t << \frac{1}{|\alpha_2|}.$$

In this case $1/\alpha t$ is the negative relative error introduced by considering Y_∞ to be the total (prompt plus delayed) correlated variance. The method leads to a result for β, if D and ϵ are known.

$$\alpha t \gtreqless 1$$

This condition presents a general treatment and analysis of Eq. (1). Rossi alpha and Y_∞ can be

[1]F. de HOFFMANN, *The Sci. and Eng. of Nuclear Power* vol. II (116/119) Addison-Wesley, (1949).
[2]E. F. BENNETT, *Nucl. Sci. Eng.*, 8 (53/61) (1960).
[3]R. W. ALBRECHT, *Nucl. Sci. Eng.*, 14, (153/158) (1962).
[4]R. P. FEYNMAN, F. de HOFFMANN and R. SERBER, *J. Nucl. Energy*, 3, (64/69) (1956).

separately measured[3,5,6] though their evaluation requires a more complicated mathematical treatment, especially when t is longer than the shortest delayed-neutron lifetime.

$\alpha t < 1$

Equation (1) may be written as follows:

$$y = \frac{V - 1}{y_\infty} = \sum_{n=2}^{n=\infty} (-1)^n \frac{(\alpha t)^{n-1}}{n!} . \qquad (3)$$

We find the third zone the most interesting and have focused our study there. Limiting the expansion to a certain value of n, one introduces a positive or negative error depending upon the parity of n. However it is not worthwhile to choose high values for n, remembering that the following expression

$$\frac{\sum_{i=1}^{F} R_i^2}{F - (n-1)}$$

(where R_i is the residue, F is the number of measurements, $n - 1$ is the number of parameters) must be minimized.

We have obtained good results for $n = 4$.

The method is successful because of its simplicity and ease of experimental application. Some advantages are:

1) With small αt's a prompt-reactor analysis is really performed, for the condition $|\alpha_2| t \ll \alpha t < 1$ conforms to Eq. (3), which ignores the presence of delayed neutrons.

2) Rossi alpha measurements are made in a region where V is varying rapidly with αt; in fact

$$\dot{y}(t = 0) = \alpha/2.$$

3) Treatment of the data is easier than for a larger range of αt, but sufficient to get α and Y_∞ with the required accuracy.

4) The total time of the measurement is really much shorter than with any other method; in this way the results are less affected by drifts in reactor power (especially when the reactor is kept critical). In fact, the condition $\alpha t < 1$ allows a maximum channel width

$$t < \frac{\tau_0}{\beta}$$

[5]A. I. MOGILNER, *Proc. Vienna Seminar*, Vol. III, (33/40) (1961).

[6]R. L. JOHNSON, Statistical Determination of Λ/β, IDO 16903, (1963).

Experimental conditions are based on an approximate evaluation of τ_0 and β.

Then the requirement that

$$c = f \epsilon t$$

(where f is the number of fissions per unit time) has a numerical value with statistical meaning must be satisfied.

This value could be in the range 10 to 100 and still gives information on variance parameters.

Remembering that

$$\bar{c} = \frac{1}{N} \Sigma_j c_j \qquad \langle (\Delta \bar{c})^2 \rangle = \frac{D}{N}$$

$$D = \frac{1}{N} \Sigma_j (c_j - \bar{c})^2 \qquad \langle (\Delta D)^2 \rangle = \frac{2(N-1)D^2}{N^2}$$

$$V = \frac{D}{\bar{c}} \qquad \langle (\Delta V)^2 \rangle \approx \frac{2D^2}{N\bar{c}^2}\left(1 + \frac{D}{2\bar{c}^2}\right)$$

are the mean values, the absolute variance, the relative variance and their variances respectively, it may be seen that experimental values of the variance have the required significance when the number gates, N, is really high ($\gtrapprox 10^4$) no matter if $\bar{c}$ is statistically small.

This short time-variance method has been applied in measuring the prompt-neutron lifetime in the organic moderated ROSPO reactor at CSN Casaccia, CNEN.

Measurements have been performed at a power level of 20 mW, giving satisfactory $\bar{c}$ values and requiring negligible corrections for counter dead time ($\approx 2\ \mu$sec).

Counts were taken with gate widths of 1 to 5 msec using as a multiscaler a LABEN 512-channel analyzer controlled by an external pulse generator.

The result thus obtained ($\tau_0 = 35\ \mu$sec) is in good agreement with earlier measurements and calculations.

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Level Separation Corrections to Resonance Integrals

Materials with complicated resonance structures are almost always present in nuclear reactors to such an extent that it is worthwhile to employ some analytic means for calculating resonance escape probability. The treatments are