

Letters to the Editors

Dysprosium Resonance Integrals

A value for the infinitely dilute absorption resonance integral of natural dysprosium, previously reported by us¹, was found to be in substantial disagreement with several subsequent determinations. The former value of $1310 \pm 220 b$, obtained from the Reactivity Measurement Facility (RMF) and Advanced Reactivity Measurement Facility-I (ARMF-I) measurements, has been changed to $1810 \pm 440 b$ after correcting an inadvertent error in the calculations. This compares favorably with the tentative value of 1970 ± 180 obtained from measurements made on a new set of samples in ARMF-II where a several-fold increase in sensitivity is realized.

A further independent check on this value was obtained from measurements of the enriched dysprosium isotopes 160-164 which gave a value of $1790 \pm 120 b$ for natural dysprosium (neglecting the contributions of Dy-156 and Dy-158). Table I lists these values along with tentative experimental values for the isotopes, and the corresponding Breit-Wigner single-level values calculated from published parameters². Relevant values reported elsewhere in the literature and compiled by McCarthy, *et al.*³, are for Dy-164, i.e., $420 \pm 50 b$ and $482 \pm 33 b$ by activation, and $406 b$ by a bound-level calculation.^a

The values were obtained from reactivity measurements of samples under 0.020-inch-thick cadmium in the swimming pool type reactors, RMF, ARMF-I, and ARMF-II, at the MTR site.

^aAdded in proof: "More recently published *ANL Reactor Constants Center Newsletter* No. 10 gives ANL calculated values for Dy-161 of 947.8b; for Dy-162 of 2610.0b; for Dy-163 of 1244.8b; for Dy-164 of 382.1b and for natural dysprosium of 1264.b."

¹J. J. SCOVILLE, E. FAST, and D. W. KNIGHT, *Trans. Am. Nucl. Soc.* 5, 337-8 (1962).

²D. J. HUGHES and R. B. SCHWARTZ, "Neutron Cross Sections," BNL-325, 2nd ed. Brookhaven National Laboratory (1958).

³A. E. MCCARTHY, *et al.*, "Neutron Resonance Integral and Age Data," *ANL Reactor Constants Center Newsletter* No. 1, (1961).

TABLE I

Dysprosium Absorption Resonance Integrals (barns)

Isotope	Measured	Calculated	Remarks
Natural	1810 ± 440	1240	RMF & ARMF-I
"	1970 ± 180		ARMF-II
"	1790 ± 120^a		"
Dy-160	1160 ± 130	none	"
Dy-161	1670 ± 170	1044	"
Dy-162	3320 ± 400	1284	"
Dy-163	1960 ± 180	1236	"
Dy-164	377 ± 34	1269	"

^aCompiled from measured isotopic values.

The samples were hollow cylinders, 0.86 inches dia., 4.25 inches long and 0.030 inches wall thickness, of Dy₂O₃ dispersed in aluminum in concentrations 1 to 10 per cent by weight. Gold specimens made and measured in a similar manner served as comparison standards.

Final values for the above measurements as well as other measurements on dysprosium and its isotopes will be submitted for publication in the near future when the work is completed.

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On the Validity of the Constant-Source Assumption for the Cell Problem

One of the most common problems in reactor design is the calculation of the thermal-flux distribution in a fuel element and its associated moderator, i.e., the cell problem. The most accurate method available for this calculation is the two-dimensional, multigroup, integral trans-

port theory code, THERMOS, written by Honeck¹. However, for repetitive design calculations a simpler and more economical (in computer time) computational scheme is required. A common practice is to use a monoenergetic treatment with a Wigner-Seitz (cylindrical) cell. This reduces the original problem with three independent variables (r, θ, E) to a much simpler one-dimensional (in r) problem, plus a spectrum calculation to compute the one-group thermal cross sections.

The source, due to slowing down, for the monoenergetic problem is usually assumed uniform in the moderator and zero in the fuel. However, due to epithermal resonance absorption in the fuel, the epithermal flux, and thus the source to thermal, will, in fact, be slightly depressed at the fuel/moderator interface relative to its magnitude at the cell edge. In this note we investigate the importance of this neglected effect on the cell disadvantage factor. A second effect which leads to a non-uniform slowing-down source, namely the non-uniform production of neutrons by fission, will be neglected in this analysis. Because of the very high energies at which fission neutrons are produced, this effect should be small compared to that of resonance absorption. Previous authors^{2,3,4,5} have shown, using age theory, that if $(4\pi\tau^2/L^2)$ is much greater than unity, then the slowing-down source is, in fact, spatially constant in the moderator. Here τ is the neutron age from the lowest resonance (6.7 eV in U^{238}) to thermal, and L is the lattice spacing. However, age theory does not apply to hydrogen moderation and the question arises as to the validity of the constant-source assumption in water-moderated systems. One would expect that in the case of hydrogen moderation the source dip at the fuel/moderator interface would be most pronounced since neutrons can be thermalized from any energy in a single collision. Thus the epithermal flux dip at each resonance will directly contribute to the thermal-source dip. We show, however, that for all practical water lattices the constant-source assumption

introduces a negligible error in the cell disadvantage factor. Before proceeding, we mention the work of Murray (reported in Reference 2) who has shown that the assumption of a constant source for a heavy-water lattice introduces a negligible error in the thermal utilization. However, several neutron collisions with deuterium are required to reduce the neutron energy from 6.7 eV (the lowest U^{238} resonance) to thermal energies, and thus one cannot use Murray's conclusions for light-water lattices.

Let us consider a simple, two-region slab lattice of fuel half-thickness x_0 and moderator half-thickness $(x_1 - x_0)$ centered at $x = 0$. We use slab geometry for simplicity, and also to avoid any possible contamination of our conclusions due to the Wigner-Seitz approximation. The source dip in a slab cell should form an overestimate of the source dip in a square cell (with cylindrical fuel element) if the distance between fuel slabs is equal to the distance between fuel rods, measured along the cell diagonal. For reasons which will shortly become obvious, we assume a chopped cosine source to the thermal group. For algebraic simplicity, we set the thermal absorption cross section of the moderator equal to zero. Calculations not reported here indicate that this simplification does not affect the conclusions. Assuming diffusion theory holds in the moderator, the equation we must then solve for the thermal flux in the moderator is

$$\frac{d^2\Phi(x)}{dx^2} = -\frac{S}{D} \cos\left(A\left[\frac{x_1 - x}{x_1 - x_0}\right]\right), \quad (1)$$

where $\Phi(x)$ is the thermal flux in the moderator, D is the thermal diffusion coefficient of the moderator, S is the constant indicating the magnitude of the thermal group source, and A is a constant governing the curvature of the thermal-group source. At $x = x_1$ one applies the usual cell condition that $d\Phi(x)/dx$ vanish. The fuel region is treated by blackness theory—i.e., an appropriate linear extrapolation distance is applied at the moderator/fuel interface, according to

$$\frac{J(x_0)}{\Phi(x_0)} = -\frac{1}{\Gamma}, \quad (2)$$

where $J(x)$ is the neutron current according to Fick's Law,

$$J(x) = -D\frac{d\Phi(x)}{dx}, \quad (3)$$

and Γ is a prescribed number characteristic of the fuel region and entering angular flux distribution into the fuel region. The solution for the disadvantage factor, δ , defined as the ratio of the average moderator flux to the average fuel flux, is, with $D = 1/3\Sigma_{tr}$,

¹H. C. HONECK, "The Calculation of the Thermal Utilization and Disadvantage Factor in Uranium Water Lattices," IAEA Conference on Light Water Lattices, Vienna (June, 1962).

²A. M. WEINBERG and E. P. WIGNER, *The Physical Theory of Neutron Chain Reactors*, p. 617, p. 648. University of Chicago Press, Chicago, (1958).

³W. A. HORNING, "A Summary of Small Source Theory Applied to Thermal Reactors", HW-34021, Hanford (1954).

⁴A. D. GALANIN, "The Thermal Coefficient in a Heterogeneous Reactor", *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy*, Vol. 5, United Nations, New York (1955).

⁵J. R. WORDEN, "Two-Group, Small Source Theory Applied to an Infinite Array of Reactor Super-Lattices", HW-63072, Hanford (1960).

$$\delta = \Sigma_{a0} x_0 \left[\Gamma + 3\Sigma_{tr}(x_1 - x_0) \left(\frac{1}{A^2} - \frac{\cot A}{A} \right) \right] , \quad (4)$$

where Σ_{tr} is the macroscopic transport cross section of the moderator and Σ_{a0} is the macroscopic absorption cross section of the fuel. In the limit of $A = 0$ (flat source), Eq. (4) reduces to the familiar result

$$\delta(A = 0) = \Sigma_{a0} x_0 [\Gamma + \Sigma_{tr}(x_1 - x_0)] . \quad (5)$$

We now define the error in δ arising from the flat-source assumption as

$$\epsilon = \frac{\delta - \delta(A = 0)}{\delta(A = 0)} . \quad (6)$$

Using Eqs. (4) and (5) in Eq. (6), we find

$$\epsilon = \frac{3 \left[\frac{1}{A^2} - \frac{\cot A}{A} \right] - 1}{1 + \frac{\Gamma}{\Sigma_{tr}(x_1 - x_0)}} . \quad (7)$$

We note from Eq. (7) that, for a given size cell and value of A , the error is a maximum when Γ is a minimum. The smallest possible value for Γ occurs if the fuel slab is a semi-infinite, purely absorbing medium. From the Milne problem, we know

$$\Gamma_{\min} = (3) (0.7104) = 2.13 . \quad (8)$$

So as not to underestimate the effect of a non-uniform source, we shall use this minimum value of Γ in our subsequent numerical calculations.

Let us now compute the spatial dependence of the slowing-down source by considering the epithermal space-energy problem. We assume diffusion theory in the moderator with P_0 energy transfer only. P_1 energy transfer is corrected for by the introduction of the transport cross section, $\sigma_{tr} = \sigma/3$ for hydrogen. We neglect moderator absorption and oxygen slowing down and take the source of neutrons to be at infinite energy. We further assume that the collision cross section of hydrogen is independent of energy. Thus the equation we must solve for the epithermal moderator flux is

$$d \frac{d^2 \phi(x, E)}{dx^2} - \sigma \phi(x, E) + \int_E^\infty \frac{\sigma}{E'} \phi(x, E') dE' = 0, \quad (9)$$

where

$$d = \frac{1}{3\sigma_{tr}} = \frac{1}{\sigma} . \quad (10)$$

Lower-case symbols in Eqs. (9) and (10) refer to the same quantities epithermally as did the corresponding upper-case symbols thermally. At $x = x_1$ one applies the usual cell condition that $d\phi(x, E)/dx = 0$. Assuming no slowing down in the fuel, this region can also be treated epithermally by blackness theory, i.e.,

$$\frac{j(x_0, E)}{\phi(x_0, E)} = -\frac{1}{\gamma(E)} . \quad (11)$$

In general, Eq. (9) cannot be solved exactly with Eq. (11) as the boundary condition. However, if $\gamma(E)$ is energy independent, Eq. (9) is easily solved by separation of variables. In view of this, let us grossly overestimate the epithermal absorption, and thus the epithermal flux dip, by replacing the fuel epithermally by a semi-infinite, purely absorbing medium for all energies. Then $\gamma(E)$ is equal to 2.13, the smallest possible value. This replacement, which may at first glance seem drastic, will nevertheless allow us to conclude that, for practical water lattices, the constant-source assumption introduces negligible error in the cell disadvantage factor.

Separating variables in Eq. (9) with $\gamma(E) = \gamma$, a constant, yields

$$\phi(x, E) = C \left[\cos(B\sigma(x_1 - x)) \right] \left[\frac{1}{E^{(1/\gamma + B^2)}} \right] , \quad (12)$$

where C is a normalization constant and B satisfies the eigenvalue equation

$$B \tan(B\sigma(x_1 - x_0)) = \frac{1}{\gamma} . \quad (13)$$

Thus, the slowing-down source to thermal is given by

$$S(x) = S \cos(B\sigma(x_1 - x)) , \quad (14)$$

where S is a normalization constant. Comparison of Eqs. (1) and (14) shows that

$$A = B\sigma(x_1 - x_0) . \quad (15)$$

Let us define the ratio of the epithermal collision cross section to the thermal transport cross section as f , i.e.,

$$f = \sigma/\Sigma_{tr} . \quad (16)$$

With Eqs. (15) and (16) the eigenvalue equation can be rewritten as

$$f \Sigma_{tr}(x_1 - x_0) = \gamma A \tan A . \quad (17)$$

Setting $\gamma = 2.13$ (so as to maximize the source dip) and $\Gamma = 2.13$ (so as to maximize the error for a given source dip), and eliminating $\Sigma_{tr}(x_1 - x_0)$ between Eqs. (7) and (17) yields

$$\epsilon = \frac{3(\tan A - A) - A^2 \tan A}{A^2 \tan A + Af} \quad (18)$$

The largest value of A possible is $A = \pi/2$, which corresponds to zero source at the moderator/fuel interface. In this limit, we find $\epsilon = 0.216$, an appreciable error. As A approaches zero, ϵ approaches $A^4/15f$. Thus in this limit the error as a function of A approaches zero extremely rapidly.

To see more clearly the effect of the various parameters on the error, we expand Eqs. (17) and (18) for small A . Eliminating A between the two expanded equations yields

$$\epsilon = \frac{f}{15\gamma^2} \Sigma_{tr}^2 (x_1 - x_0)^2 \quad (19)$$

We note that the error is proportional to the square of the cell size for small cells (for which Eq. (19) is valid).

For the purpose of making numerical comparisons, we need to assign values to f , γ , and Σ_{tr} . An experimental value for Σ_{tr} for cold water is $\Sigma_{tr} = 2.35 \text{ cm}^{-1}$ (Ref. 6) which corresponds to a microscopic transport cross section for water of 70 barns. For σ , the macroscopic epithermal collision cross section, a proper choice would be an energy-averaged value over the resonance-energy region. So as not to underestimate ϵ , we take the microscopic collision cross section of hydrogen as 21 barns, the largest value in the resonance region (corresponding to $E = 6.7 \text{ eV}$). There is some question whether or not to include the oxygen cross section in the calculation of σ . Since the oxygen slowing down was neglected in the analysis, a consistent treatment would be not to include it here. However, we shall include oxygen scattering of 4 barns to assure ourselves that we are not underestimating the error. The above discussion then leads to an epithermal microscopic collision cross section for water of 46 barns and thus $f = 0.66$. For reasons already mentioned, we set $\gamma = 2.13$.

Let us now compute the error from Eqs. (17) and (18) for typical light-water cells. From the published data⁷ on Dresden (boiling-water reactor) one can compute 0.676 cm as the distance from the

cell boundary to the surface of the fuel/clad assembly measured along the diagonal of the cell. The analogous figure⁸ for Yankee (pressurized-water reactor) is 0.325 cm. With $\Sigma_{tr}(x_1 - x_0) = (0.676)(2.35) = 1.59$, Eqs. (17) and (18) yield $\epsilon = 0.0124$. The error for the Yankee cell is much less (about a factor of four) due to its smaller size. Thus our analysis shows that, for typical water lattices, an overestimate of the error in the disadvantage factor due to the assumption of a spatially constant source is of the order of one percent, which results in a few tenths percent error in the thermal utilization (and thus the reactivity).

An error of a few tenths percent in the utilization is small from a design standpoint due to other errors which are implicit (and more significant) in a design cell calculation (Wigner-Seitz approximation, basic cross-section data, separability of space and energy, etc.). Thus we conclude that, for design calculations of practical light-water systems, the assumption of a spatially constant slowing-down source in the moderator introduces a small error in the calculated reactivity. Further, since our analysis has *grossly* overestimated the error in the disadvantage factor, one would suppose that the actual error in the thermal utilization is probably an order of magnitude less than that calculated here (of the order of hundredths of a percent). Said another way, the actual error in the utilization is probably negligibly small for cells that are much larger than those used in practical light-water reactors.

As a final note, let us compute the source depression at the fuel/moderator interface relative to the source strength at the cell edge for the Dresden cell. We find

$$\frac{\hat{S}(x_1) - S(x_0)}{S(x_1)} = 0.203. \quad (20)$$

We see that a relatively large source dip leads to a small error in the cell disadvantage factor.

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⁶S. GLASSTONE and M. C. EDLUND, *The Elements of Nuclear Reactor Theory*, p. 127. D. Van Nostrand, New York, (1952).

⁷F. A. HOLLENBACK, *Nucleonics*, 17, No. 12, 65-75 (1959).

⁸R. J. COE, G. A. REED and D. E. VANDENBURGH, *Nucleonics*, 19, No. 3, 53-61 (1961).