lated. The average beta energy determined here agrees well with the value of 0.0704 ± 0.0040 MeV which Hovi and Niemela[3] determined by a calorimetric method.

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The Infinite Dilution Resonance Integral of Thorium-232

Reported values of the infinite dilution resonance integral of Th^{232} have covered an unusually wide range. Measurements have ranged from 67 to 106 barns, while values calculated from resonance parameters have varied from 76 to 97 barns. Most recently, Brose[1] has reported an experimental value of 82.7 ± 1.8 barns.

In connection with a study of U^{238}/Th^{232} mixtures[2], we have also measured and calculated the thorium infinite dilution resonance integral. The results, which afford an interesting comparison with previous work, are:

Measured:	81.2 ± 3.4 barns
Calculated:	82.3 barns.

The agreement with Brose's value is very good, especially in light of the fact that the experimental methods were different. Brose used an activation technique, while we used a static reactivity technique.

Measurements were made in the Advanced Reactivity Measurement Facility (ARMF-II) at the National Reactor Testing Station. The samples, containing a wide range of ThO_2 concentrations, were prepared by compacting powders in identical aluminum cylinders approximately one inch in diam and five inches long. To control absorber density in the different samples, lead dioxide powder was used as a diluent. The sample of lowest absorber density contained approximately 0.25 wt% thorium dioxide. Absorber particle size was sufficiently small to make particle self-shielding negligible.

Calibration was effected by performing both activation and reactivity measurements on gold samples; the standard used was a value of 1579 barns for the gold infinite dilution integral from a low-energy cutoff of 0.5 eV. This value was calculated from the recent Columbia data[3]. Experimentally based corrections to the observed reactivity data were made to account for the effects of the aluminum capsules, scattering by the absorber and the diluent, and absorption in the diluent. Theoretical corrections were applied to correct for the effect of the 20-mil cadmium filter, slight deviation of the ARMF flux from a $1/E$ spectrum throughout the resonance region, energy dependence of the adjoint flux, and the reactivity effects of fast fissions. The latter three corrections were based on a transport calculation of the real and adjoint fluxes in the cadmium-shielded measurement position.

The infinite dilution resonance integral was determined by extrapolating an analytical fit of the reactivity worths to yield a value of reactivity for each absorber atom at zero absorber concentration in the sample; this quantity is proportional to the infinite dilution absorption integral. An expression of the form

$$\rho_a = A(1 - e^{-BN})$$

where

ρ_a = reactivity due to absorption

N = absorber atom density

A and B = constants determined by the fit

gave an excellent fit to the data. The resulting infinite dilution resonance integral given above is adjusted to a cutoff energy of 0.5 eV and includes the $1/v$ contribution. One barn of the 3.4 barn uncertainty is due to a 3% uncertainty assigned to the gold infinite dilution value.

The calculated value given above is based on a 'best' set of resonance parameters compiled recently by the Cross Section Evaluation Center at Brookhaven National Laboratory[4]. The calculation

[1] M. BROSE, *Nucl. Sci. Eng.*, **19**, 244 (1964).

[2] W. K. FOELL, "Resonance Absorption of Neutrons in Mixtures of Thorium-232 and Uranium-238: An Investigation of Interference Between Absorbers," Stanford University Doctoral Thesis, (1964).

[3] J. S. DESJARDINS, J. L. ROSEN, W. W. HAVENS, Jr. and J. RAINWATER, *Phys. Rev.*, **120**, 2214 (1960).

[4] J. STEHN, Cross Section Evaluation Center, Brookhaven National Laboratory, private communication, (March, 1964).

used resolved parameters up to an energy of 4 keV, with an average gamma width of 0.0231 eV. The range between 4 keV and 32 keV was treated on a statistical basis. The contributions of each range are:

Energy (eV)	Resonance Integral (barns)
0.5 - 19.6	1.8
19.6 - 4060	77.7
4060 - 32000	0.7
Above 32000	1.1
p-wave capture	1.0
	82.3

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Statistical-Error Estimation for the Transfer-Function Measurements of a Noisy Reactor System

The application of the cross-correlation method in the determination of the dynamic response of a system which contains extraneous noise has long been accepted as one of the most reliable methods for recovering signals in the presence of noise. The theory of this method can be found in the literature[1,2,3]. Also, many discussions have been presented on the statistical errors of the results of measurements where some discrepancies appear because of the limitation in application of the theory[2,4]. In many situations, intuition plays a major role in estimating these statistical errors. Under the condition of using a sinusoidal input signal for transfer-function measurements, for a noisy reactor (e.g. EBWR) intuition has often led to the simple conclusion that the statistical error is dependent soley on the length of the record. In order to derive a more correct estimate of the error, the characteristics of the system noise and signal must be included in estimating the error.

The result of an investigation using statistical theory is presented here. The two most common types of noise have been used for illustration.

The assumptions that have been made are 1) the system noise is a stationary random process; 2) the ergodic hypothesis is valid; 3) the statistical error has a Gaussian distribution.

Let the system input be x, where

$$x = A \sin \omega t \ , \tag{1}$$

and let the system output be y, where

$$y = B \sin (\omega t + \phi) + n(t) \ . \tag{2}$$

The finite-time cross correlation between x and y is

$$\phi_{yx}(\tau) = \frac{1}{T}\int_0^T \Big[A \sin(\omega t + \omega\tau)\Big]\Big[B \sin(\omega t + \phi) + n(t)\Big]dt$$

$$= \frac{AB}{2} \cos(\omega\tau - \phi) + \epsilon(\tau, T) \ , \tag{3}$$

where $\epsilon(\tau, T)$ is the error function, and

$$\epsilon(\tau, T) = \frac{1}{T}\int_0^T A \sin(\omega t + \omega\tau) n(t)\, dt \ . \tag{4}$$

The standard deviation of ϵ can be found from the fourth moment[2] of functions x and n where

$$\epsilon^2(\tau, T) = \frac{2}{T^2}\int_0^T (T - \nu) \times$$

$$\times \Big[\Phi_{xx}(\nu)\, \Phi_{nn}(\nu) + \Phi_{xn}(\nu + \tau)\Phi_{nx}(\tau - \nu)\Big]d\nu$$

$$T = \frac{2k\pi}{\omega} \quad k = 0, 1 \ldots , \tag{5}$$

where $\Phi_{xx}(\nu)$, $\Phi_{nn}(\nu)$, $\Phi_{xn}(\nu)$ and $\Phi_{nx}(\nu)$ are the theoretical correlation functions for x and n:

$$\Phi_{xn}(\nu) = \Phi_{nx}(\nu) = \lim_{T \to \infty} \frac{1}{T}\int_0^T \times$$

$$\times \Big[A \sin \omega t\Big]\Big[n(t + \nu)\Big]dt = 0 \tag{6}$$

$$\Phi_{xx}(\nu) = \lim_{T \to \infty} \frac{1}{T}\int_0^T A^2 \sin \omega t \sin(\omega t + \omega\nu)\, dt$$

$$= \frac{A^2}{2} \cos\omega\nu \tag{7}$$

$$\Phi_{nn}(\nu) = \lim_{T \to \infty} \frac{1}{T}\int_0^T n(t)\, n(t + \nu)\, dt$$

$$= \sigma_n^2 \ \Phi'_{nn}(\nu) \ , \tag{8}$$

[1] Y. W. LEE, *Statistical Theory of Communication*, John Wiley and Sons, Inc., New York, (1960).

[2] J. S. BENDAT, *Principles and Applications of Random Noise Theory*, John Wiley and Sons, Inc., New York, (1958).

[3] W. B. DAVENPORT and D. L. ROOT, *An Introduction to Theory of Random Signals and Noise*, McGraw-Hill Book Co., New York, (1958).

[4] V. RAJAGOPAL, "Experimental Study of Nuclear Reactor Internal Noise and Transfer Function Using Random Reactivity Variations and Correlation Analysis," (microfilm), University of Michigan, Ann Arbor, (1961).