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Letters to the Editors

Two-Group Material Bucklings in a Two-Region System

In a one-region system, criticality cannot be attained unless the material-buckling equation has a positive root. In a two-region system this condition is no longer necessary.

Strangely, this surprises many reactor experts. Familiarity with ν^2, which is always negative, and μ^2, which is positive if a one-region system is to be critical, obscures the possibility of a system being critical if μ^2 is also negative. Nevertheless, if both μ^2 and ν^2 are negative in a fuel-bearing region, the addition of a non-fueled second region with only negative bucklings can produce a critical system.

A long cylinder of natural uranium surrounded by a moderator is a familiar example of such a system. The flux in the uranium is described by two Bessel functions of the same type. If the fueled region is a slab, its flux is described by two hyperbolic cosines.

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On the $(K\beta/\ell)$ Pulsing Theory*

In the theory of pulsed-neutron source measurements for the $(k\beta/\ell)$ technique,[1] the analytical model was based on a bare monoenergetic diffusion-theory model with m-delayed precursors. Following the eigenfunction expansion methods as outlined by Cohen,[2] it is a simple matter to show that the monoenergetic diffusion-theory approximation can be replaced by the monoenergetic transport-theory approximation. This then places the $(k\beta/\ell)$ technique on a more realistic basis. The $(k\beta/\ell)$ technique depends merely upon the properties of the inhour equation and, since this equation is invariant with respect to the production and destruction operators (diffusion and transport theory), the above result is not particularly surprising.

Consider the time-dependent angular neutron density, $N(\vec{r},\vec{\Omega};\vec{r'}t)$, at some point r for neutrons in the direction $\vec{\Omega}$ due to a unit isotropic source at $\vec{r'}$ at time t. The monoenergetic transport equation for neutrons of speed v and precursor equations are given by

$$\partial N/\partial t+v\vec{\Omega}\cdot\nabla N+\sigma vN=\int vN(\vec{r},\vec{\Omega'};\vec{r'},t)\sigma(\vec{\Omega'}\to\vec{\Omega},\vec{r})d\vec{\Omega'}$$

$$+(1/4\pi)\,v\nu(1-\beta)\sigma_f\tilde{N}+(1/4\pi)\sum_{i=1}^{m}\lambda_iC_i(\vec{r},t)+S(\vec{r},t)\quad(1)$$

$$\partial C_i(\vec{r},t)/\partial t=-\lambda_iC_i+\beta_iv\nu\sigma_f\tilde{N},\quad i=1,2,\ldots,m\quad(2)$$

where $S(\vec{r},t)$ represents the source, $\tilde{N}$ represents the total density, i.e.,

$$\tilde{N}=\int N(\vec{r},\vec{\Omega};\vec{r'},t)\,d\vec{\Omega}\quad(2a)$$

and the remaining symbols have their usual meaning. For a pulsed source, pulsed at the rate of R pulses per second,

$$S(\vec{r},t)=\frac{1}{4\pi}\,\delta(\vec{r}-\vec{r'})\sum_{n=0}^{\infty}\delta\left(t-\frac{n}{R}\right)\quad(3)$$

where the δ's represent the usual Dirac delta functions. It should be noted that we are assuming a uniform pulse strength, equal number of neutrons per burst; this is merely for convenience and not necessary to the development.[3] Following Cohen we introduce the reactivity eigenfunctions, $\psi_s(\vec{r},\vec{\Omega})$, as defined by

$$v\vec{\Omega}\cdot\nabla\psi_s(\vec{r},\vec{\Omega})+\sigma v\psi_s-\int v\psi_s(\vec{r},\vec{\Omega'})\sigma(\vec{\Omega'}\to\vec{\Omega},\vec{r})d\vec{\Omega'}$$

$$=(1/4\pi)(v\nu\sigma_f/k_s)\tilde{\psi}_s\ ,\quad(4)$$

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[1]E. GARELIS and J. L. RUSSELL, Jr., *Nucl. Sci. Eng.*, 16, 263-270 (1963).

[2]E. R. COHEN, *Proc. 2nd Intern. Conf. Peaceful Uses of Atomic Energy*, Geneva, (1958), paper 629.

[3]E. GARELIS, *Nucl. Sci. Eng.*, 18, 242 (1964).