lation distance, z_0, as calculated from the exact analysis are given in Table II.

We feel that the type of comparison made above is important in verifying the accuracy of any numerical code that solves the transport equation. For the DTF code, the handling of the boundary conditions, the eigenvalue search, and the linear and P_2 scattering options is apparently accurate. Although no information is obtained about material spatial variation or multigroup treatment, such comparisons can be made, albeit with very complicated exact solutions. The table provides solutions against which other codes can be compared, and in addition contains useful information about the behavior of critical thickness for anisotropic scattering. Even for the simple monoenergetic, homogeneous case, additional meaningful comparisons can be made. Exact solutions for critical radii for one-dimensional spheres can be obtained with relatively minor changes in the slab critical equation, and such solutions could be used to examine the treatment of ray-to-ray transfers (streaming) in curved geometry. Mitsis[2] has given an exact critical equation for cylindrical geometry, solutions to which could be used to investigate the accuracy of two-dimensional angular quadrature. Although, in themselves, such comparisons verify only parts of ofttimes extremely complex codes, they provide the foundation upon which confident numerical computing can be based.

TABLE II

Extrapolation Distance, z_0 (mfp)

$c + c'$	c	$(z_0)_{P_1}$	$(z_0)_{P_2}$
1.05	0.1	0.72414	0.72394
	0.3	0.84222	0.84151
	0.5	1.00594	1.00456
	0.7	1.24815	1.24580
	0.9	1.64322	1.63928
1.1	0.1	0.69042	0.68982
	0.3	0.80043	0.79826
	0.5	0.95181	0.94735
	0.7	1.17336	1.16537
	0.9	1.52899	1.51498
1.2	0.1	0.63139	0.63013
	0.3	0.72719	0.72252
	0.5	0.85747	0.84753
	0.7	1.04518	1.02674
	0.9	1.34000	1.30673
1.3	0.1	0.58146	0.57974
	0.3	0.66524	0.65872
	0.5	0.77811	0.76395
	0.7	0.93889	0.91204
	0.9	1.18783	...[a]
1.4	0.1	0.53873	0.53670
	0.3	0.61232	0.60452
	0.5	0.71061	0.69349
	0.7	0.84928	0.81635
	0.9	1.06161	...[a]

[a]Not calculable by present exact program due to the appearance of a second discrete eigenvalue.

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A Note on the Inverse Kinetics Analysis

In their article[1] Murray, Bingham and Martin have presented some solutions of the reactor kinetics equations for the reactivity variation required to achieve specified power responses. They show the usefulness of such an inverse method and demonstrate it by several examples. It is felt that an important case could be added to the list of power functions considered in the above-mentioned paper.

For the reactor power function of the type

$$n = n_0 \exp(\alpha t)\,(1 + A \sin\omega t)$$

the corresponding reactivity function can be found by solving reactor point kinetics equations. This is

$$\rho = \alpha\left(\ell^* + \sum_i \frac{\beta_i}{\lambda_i + \alpha}\right) + \sum_i \frac{\beta_i \lambda_i \omega^2}{(\lambda_i + \alpha)^3 + \omega^2(\lambda_i + \alpha)} \times$$

$$\times \frac{A \sin\omega t}{1 + A \sin\omega t} + \left[\omega \ell^* + \sum_i \frac{\beta_i \lambda_i \omega}{(\lambda_i + \alpha)^2 + \omega^2}\right] \times$$

$$\times \frac{A \cos\omega t}{1 + A \sin\omega t} - \frac{Q\ell^* \exp(-\alpha t)}{n_0\,(1 + A \sin\omega t)} -$$

$$- \sum_i \left[\frac{\lambda_i \ell^* C_{i0}}{n_0} - \frac{\beta_i \lambda_i}{\alpha + \lambda_i} + \frac{A \beta_i \lambda_i \omega}{(\lambda_i + \alpha)^2 + \omega^2}\right] \times$$

$$\times \frac{\exp(-\alpha t - \lambda_i t)}{1 + A \sin\omega t},$$

where C_{i0} is the initial delayed-emitter concentration and Q characterizes the external neutron source. The other symbols have their usual meaning.

This reactivity function has been used[2] in defining the equivalent transfer functions of a reactor for both the critical state and the sub- and supercritical state in dependence on the value α. The nonlinear effects (the influence of the amplitude A) and the conditions for using the above-mentioned transfer functions have been also evaluated.

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[1]R. L. MURRAY, C. R. BINGHAM and Ch. F. MARTIN, "Reactor Kinetics Analysis by an Inverse Method," *Nucl. Sci. Eng.*, **18**, 481-490 (1964).

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On the Green's Function of Monoenergetic Neutron Transport Theory*

In recent years several authors (Refs. 1, 2, 3 among others) have used the normal mode approach to the solution of the monoenergetic neutron transport equation. Each author has presented a development of the angular Green's function. We shall illuminate here several misleading aspects which are generated by these previous discussions.

For the sake of brevity, let us consider the case of isotropic scattering in a medium with plane symmetry. In the notation of Ref. 2, the neutron flux resulting from the 'monodirectional' source $\delta(\mu - \mu_0)\,\delta(x)$ is given by

$$\rho_I(x) = \frac{\phi(L,\mu_0)}{M_+} e^{-x/L} + \int_0^{+1} \frac{\phi(\nu,\mu_0)}{M(\nu)} e^{-x/\nu}\, d\nu, \quad x > 0$$

$$= -\frac{\phi(-L,\mu_0)}{M_-} e^{x/L} - \int_{-1}^{0} \frac{\phi(\nu,\mu_0)}{M(\nu)} e^{-x/\nu}\, d\nu, \quad x < 0. \tag{1}$$

It should be noted that $\rho_I(x)$ has no unusual functional properties since the uncollided neutron contribution does not appear as a δ distribution. Thus, it is not surprising that Eq. (1) is consistent with previously published results based on a normal mode expansion. However, Eq. (1) can be derived using the elementary methods of Ref. 4.

Using Eq. (1) we can determine the angular Green's function (i.e. the angular flux resulting from the present source) via the relation

$$\Psi_G(x,\mu;\mu_0) = \Psi_0(x,\mu;\mu_0) + \frac{c}{2}\int_{-\infty}^{+\infty} \frac{e^{-|x-x'|/|\mu|}}{|\mu|}\rho_I(x')dx', \tag{2}$$

where $\Psi_0(x,\mu;\mu_0)$ is the uncollided angular flux and c is the scattering probability. It is a straightforward matter to reduce Eq. (2) to the form

$$\Psi_G(x,\mu;\mu_0) = \frac{\phi(L,\mu)\phi(L,\mu_0)}{M_+} e^{-x/L} + \int_0^{+1} \frac{\phi(\nu,\mu)\phi(\nu,\mu_0)}{M(\nu)} e^{-x/\nu}\, d\nu + h(\mu)\Delta(\mu,\mu_0)e^{-x/\mu}, \quad x > 0 \tag{3}$$

$$= -\frac{\phi(-L,\mu)\phi(-L,\mu_0)}{M_-} e^{x/L} - \int_{-1}^{0} \frac{\phi(\nu,\mu)\phi(\nu,\mu_0)}{M(\nu)} e^{-x/\nu}\, d\nu - h(-\mu)\Delta(\mu,\mu_0)e^{-x/\mu}, \quad x < 0,$$

where

$$\Delta(\mu,\mu_0) = \frac{\delta(\mu - \mu_0)}{\mu_0} - \frac{\phi(L,\mu)\phi(L,\mu_0)}{M_+} - \frac{\phi(-L,\mu)\phi(-L,\mu_0)}{M_-} - \int_{-1}^{+1} \frac{\phi(\nu,\mu)\phi(\nu,\mu_0)}{M(\nu)} d\nu$$

and $h(\mu)$ is the unit step function (i.e., $h(\mu) = 0$, $\mu < 0$, and $h(\mu) = 1$, $\mu \geq 0$).

The previously reported angular Green's functions have taken the form of Eq. (3) but with $\Delta(\mu,\mu_0) = 0$. Essentially, these previous developments are based on the closure condition for the function set $\{\phi(\pm L, \mu), \phi(\nu,\mu)\}$,

$$\delta(\mu - \mu') = \frac{\mu\phi(L,\mu)\phi(L,\mu')}{M_+} + \frac{\mu\phi(-L,\mu)\phi(-L,\mu')}{M_-} + \mu\int_{-1}^{+1} \frac{\phi(\nu,\mu)\phi(\nu,\mu')}{M(\nu)} d\nu. \tag{4}$$

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