5-7 μm for other specimens. Sintering in CO_2 did not lead to any carbon pickup[4], so the overall carbon content in any one of the sintered pellets should have been less than 100 parts/10^6, as usual.

The results of the fission-gas release experiments are listed in Table II.

The release rate parameters obtained by us at 1400°C are less or of the same order of magnitude as those obtained in earlier work by other investigators (see, for instance, Refs. 13 and 14). It should be noticed that this might be due to the somewhat higher irradiation dose adopted by us and the consequent decrease of the D' coefficient[15].

The reproducibility of data from duplicate specimens recorded in Table II is satisfactory. The release rates from pellets made by other techniques are never significantly higher than those of pellets made by the usual hydrogen-sintering method. On the contrary, the pellets sintered in carbon dioxide seem to have a better fission-gas retention capacity, even though we are inclined to feel that this feature is due to chance. Fission-gas release mechanisms have not yet been identified with certainty. The thermal diffusion process, however, is no doubt responsible for a substantial fraction of the total release. The extent to which this mechanism is affected by the structural conditions and purity of the specimens is not known. Although further work is required to test the fission-gas behavior in operating conditions in a power reactor, it is believed that our preliminary results substantiate the use of pellets made with one of the unconventional methods, if this method leads to economic gain.

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Large Reactor Excursions Starting at Operating Conditions

The present note is an extension of previous work[1] on large excursions in reactors, to include those starting at operating, steady-state conditions. The previous work was limited to excursions starting at zero power.

Consider a *critical* reactor whose power, power due to delayed neutron precursors, and temperature are $(n,C,T) = (n_0,C_0,T_0)$. This steady state is characterized by the constant control reactivity, which balances exactly the reactor reactivity feedback, $\rho_c = -f(T_0)$, where $f(T)$ is the temperature-dependent reactivity feedback. If at a given instant, $t = 0$, a large reactivity step is inserted and all coolant is lost, the energy released in such an excursion is contained entirely in the core, and the reactor is described by the following kinetic equations[1]:

$$\frac{dn}{dt} = \frac{\rho_0 - \beta + f(T)}{\ell} n + \lambda C$$

$$\frac{dC}{dt} = \frac{\beta}{\ell} n - \lambda C \qquad (1)$$

$$\frac{dT}{dt} = \gamma n.$$

With the above initial conditions, at $t = 0$, the total reactivity equals the step insertion ρ_0 and $(n,C,T) = (n_0,C_0,T_0)$. The notation of system (1) is given in Ref. 1.

The first integral of system (1) found in Ref. 1 reads

$$n + C - \frac{\rho_0}{\gamma\ell} T - \frac{1}{\gamma\ell} \int f(T)dT = A. \qquad (2)$$

Here A is an arbitrary constant giving the initial condition. In our case this is

$$(n,C,T) = (n_0,C_0,T_0),$$

so that

$$A = \left(1 + \frac{\beta}{\lambda\ell}\right) n_0 - \frac{1}{\gamma\ell} \left[\int f(T)dT\right]_{T=T_0}, \qquad (3)$$

noting that $C_0 = \frac{\beta}{\lambda\ell} n_0$.

From here and using Eq. (3), one can follow exactly the treatment of Ref. 1 to obtain *all* the results corresponding to the general ones presented there for excursions starting at shutdown, where the initial condition was $(n,C,T) \approx (0,0,0)$ and

$$A = -\frac{1}{\gamma\ell} \left[\int f(T)dT\right]_{T=0}.$$

To illustrate this fully we have carried through the procedure in the simplest case where $f(T) = -a_1(T-T_0)$ (constant temperature coefficient). Only in this linear case, it is possible and convenient to translate the temperature scale by T_0, so that $f(T) = -a_1 T$ and the temperatures calculated from Eq. (1) are temperature increases over the initial one,

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T_0. (In most cases the reactivity feedback $f(T)$ is associated with the nuclear Doppler effect and T is then the absolute temperature of the core.)

The results obtained are given together with the corresponding ones of Ref. 1. The final (maximum) temperature reached by the reactor in the excursion is

$$T_{max} = \frac{\rho_0 + \sqrt{\rho_0^2 + 2a_1\gamma\ell(1+\beta/\lambda\ell)n_0}}{a_1} \qquad (4)$$

In the case of excursions starting at shutdown, we had

$$T_{max} = \frac{2\rho_0}{a_1}. \qquad (4a)$$

To first order in $\epsilon[\epsilon = \lambda\ell$, see Ref. 1], the coordinates of the peak of the trajectory, giving the maximum power and the temperature at maximum power are

$$T_c = \frac{B}{a_1}, \; n_{max} = \frac{B^2}{2\gamma a_1 \ell}. \qquad (5)$$

Here $B = \rho_0 - \beta$ is the prompt reactivity insertion. One sees that (except for relative errors of order ϵ) the peak of the trajectory has the same coordinates as in Ref. 1. For clarity, in Fig. 1 we give the trajectories in the dimensionless phase plane (T,n) of two large excursions, one starting at shut down ($n_0 \approx 0$) and the other at an initial power n_0. Figure 1 shows that the trajectories are the same (except for relative differences of order ϵ) in the first part of the excursion. Physically, in a large prompt excursion the neutron multiplication is so large that the neutron population at the peak is very much larger than the initial one and practically independent of it. However, the reservoir of delayed neutron precursors is considerably larger for an excursion starting at an operating power n_0 (note that $C_0 = \frac{\beta}{\lambda\ell} n_0 \approx 10^3 n_0$—for example in SPERT I (see Ref. 1), $\beta/\epsilon = \beta/\lambda\ell = 3.26 \times 10^3$) than at shut down, $n_0 \approx 0$; this is shown clearly in the second part of the trajectories, which are in fact the tails[1] due to the delayed neutrons. As the term $2\beta n_0$ appearing in the radicand of the abscissa of the shut down point in Fig. 1 is a constant for a given operating power, one sees that its importance relative to ρ_0^2 decreases with increasing ρ_0. In the limit of very large ρ_0, the two shut down points coalesce since $2\beta n_0 \ll \rho_0^2$. However, for even quite large reactivity insertions, the difference between the (maximum) shut down temperature increases given by Eqs. (4a) and (4) is quite significant, as it is shown in the example of Table I. Here we consider a reactor of the following characteristics:

$a_1 = 10^{-3}(\text{degC})^{-1}$, $\ell = 10^{-4}$sec, $\beta = 0.0074$,

$\lambda = 10^{-1}\text{sec}^{-1}$, $C = \frac{1}{\gamma} = 200$

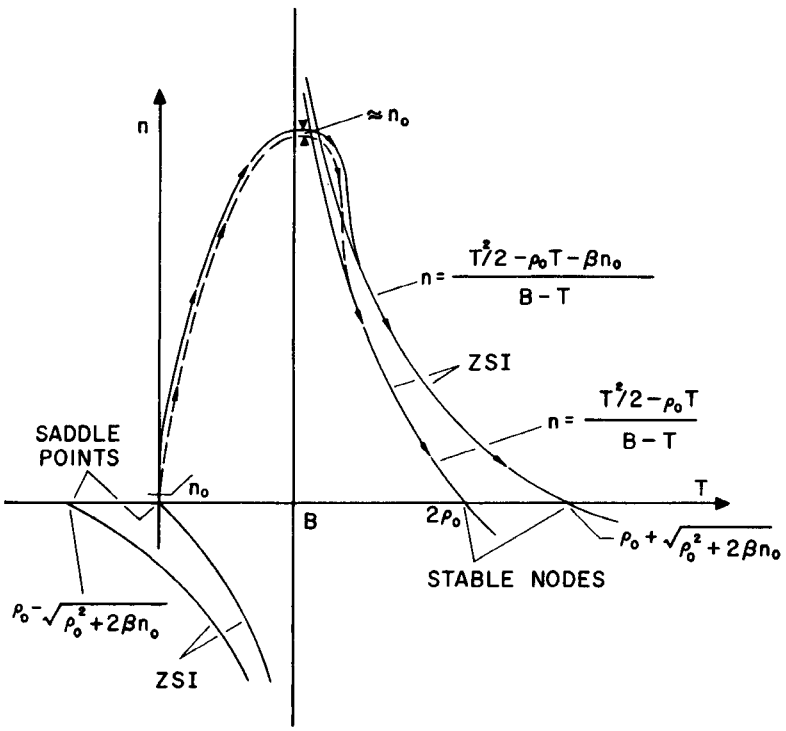

Fig. 1. Trajectories in the (T,n) phase plane for large prompt excursions starting at shutdown (dash curve) and at operating conditions (solid curve). The significance of the zero-slope isocline (ZSI) curves is discussed in Ref. 1.

TABLE I

Excursions after Step Reactivity Insertions

Starting at Shut Down			Starting at Operating Power and Temperature			Relative Difference in T_{max}
ρ_0 (β=0.0074)	n_{max} (kW)	T_{max}[a] (°C)	n_{max}	$\frac{n_{max}}{n_0}$	T_{max}[b]	%
10^{-2}	6 760	20	6 960	33.8	25.7	22.5
1.5×10^{-2}	57 800	30	58 000	289	34.3	12.5
2.0×10^{-2}	159 000	40	159 200	795	43.4	7.8

[a]Increase over room temperature.
[b]Increase over operating temperature.

kW-sec(deg C)$^{-1}$ (heat capacity) and n_0 = 200 kW. It must be recalled[1] that when $B = \rho_0 - \beta > 0$ is not large enough, the peak power calculated using Eq. (5) is no longer adequate. However, the final (maximum) temperature reached in the excursion is always given by Eq. (4), and the difference in the values of T_{max} calculated using Eqs. (4) and (4a) becomes very large.

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Comparisons of Exact and S_N Solutions of the Monoenergetic Critical Equation with Anisotropic Scattering

One of the useful applications of the method of singular integral equations[1,2,3] is to provide exact solutions of the Boltzmann transport equation against which numerical code solutions can be compared. We have made such a comparison by calculating the critical thickness of a plane homogeneous slab in the monoenergetic approximation, considering the transport equation

$$\mu \frac{\partial \Psi(x,\mu)}{\partial x} + \Sigma_t \Psi(x,\mu) = \left(\frac{\Sigma_s^{iso} + \nu\Sigma_f}{2}\right) \int_{-1}^{1} \Psi(x,\mu')d\mu' + \Sigma_s^{aniso} \sum_{n=0}^{2} \frac{2n+1}{2} b_n P_n(\mu) \int_{-1}^{1} P_n(\mu')\Psi(x,\mu')d\mu', \tag{1}$$

subject to the boundary conditions

$$\Psi(t/2,\mu) = 0 \qquad \mu < 0$$
$$\Psi(-t/2,\mu) = 0 \qquad \mu > 0. \tag{2}$$

In Eq. (1) the anisotropic scatterer is represented by a three-term Legendre expansion in a form that maintains the anisotropic-scattering cross section as a separate free parameter. Truncation of the expansion is justified either on the grounds that the expansion coefficients become small or on the grounds that the angular flux is relatively isotropic. We have solved Eqs. (1) and (2) for several values of the secondaries ratio $c + c'$ where c is the anisotropic-scattering ratio

$$c = \Sigma_s^{aniso}/\Sigma_t \tag{3}$$

and c' is the isotropic secondaries ratio

$$c' = (\Sigma_s^{iso} + \nu\Sigma_f)/\Sigma_t . \tag{4}$$

For each value of $c + c'$ we have varied c to observe the effect of increasing anisotropy of scattering. For the purpose of calculation the b_n of elastic hydrogen scattering were used ($b_0 = 1$, $b_1 = \frac{2}{3}$, $b_2 = \frac{1}{4}$), but no restriction on the calculations is implied as long as the truncation is justified. For elastic hydrogen scattering $b_3 = 0$ and $b_4 = -\frac{1}{24}$. For these coefficients no appreciable

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