Letter to the Editor

On the Calculation of Neutron Lifetimes

Kiefhaber's note[1] on the calculation of neutron lifetimes reopens an interesting topic. The first point I would make is that the discrepancy is not so much between measured and calculated values of the neutron lifetime (generation time) $\bar{\Lambda}$, but rather in the ratio $\bar{\beta}/\bar{\Lambda}$, for it is this that is measured, and this ratio, therefore, should then be computed with maximum accuracy. In going along for the rest of this Letter with Kiefhaber's assumption that only the ratio $\bar{\Lambda}$ is fundamental, we must not lose sight that we must, in practice, set up a computation for $\bar{\beta}/\bar{\Lambda}$. However, for the purposes of exposition, I readily agree that $\bar{\Lambda}$ is of interest and enables one to talk in an elementary two-group model where a three-group model is the simplest model that could represent the physics of a computation of $\bar{\beta}/\bar{\Lambda}$.

While agreeing with the sentiment that a bilinear homogeneous functional of flux and importance, such as the generation time

$$\bar{\Lambda} = \int \phi^+ v^{-1} \phi \, d\boldsymbol{\nu} \Big/ \int \phi^+ \nu \Sigma^f \phi \, d\boldsymbol{\nu} \quad ; \quad M^T\phi^+ = 0 = M\phi \tag{1}$$

in an obvious matrix notation for multigroup theory, should be computed in a few-group approximation by a variational reduction of the group constants, there is still the question, unspoken by Kiefhaber, of how these give rise to weighting functions. That is, if we require $\bar{\Lambda}$ to be insensitive to approximations in ϕ^+ and ϕ which might be associated with a group reduction approximation, we should employ a generalized variational principle for such a homogeneous bilinear ratio of the Pomraning[2] type where $\bar{\Lambda}$, itself, leads to source terms in a generalized flux and a generalized adjoint equation whose field solutions provide the required weighting terms.

I have given an example of this procedure in a note[3] on generalized perturbation theory in the two-group model so that I need only quote the results here. Writing $\alpha_2(1 + L_f^2 B^2)\, p\Sigma_f/\Sigma_t$, the ratio of fast-to-thermal flux in a bare reactor, and $\alpha^+ = p/(1 + L_f^2 B^2)$ as the ratio of fast-to-thermal importance in the same model, we may show that the generalized functions appropriate to computing the generation time have an energy dependence

$$M^T\psi^+ + \frac{\partial\bar{\Lambda}}{\partial\phi} = 0 = M\psi + \frac{\partial\bar{\Lambda}}{\partial\phi^+}$$

$$\phi^+ = \begin{pmatrix}\alpha^+\\1\end{pmatrix} \quad ; \quad \psi^+ = \begin{pmatrix}0\\1\end{pmatrix} \quad : \quad \phi = \begin{pmatrix}\alpha\\1\end{pmatrix} \quad ; \quad \psi = \begin{pmatrix}1\\0\end{pmatrix} \tag{2}$$

and the differences are obvious.

Using the variational method to reduce two-group equations to one group, we obtain different answers if we pose different questions.

1. Best criticality approximation:

$$(\phi^+ M\phi)\,\phi(\boldsymbol{\nu}) = 0 \equiv (\phi M^T\phi^+)\,\phi^+(\boldsymbol{\nu}) = 0 \tag{3}$$

Result: self adjoint equation

$$[(\alpha^+\alpha D_f + D_t)\,\nabla^2 - (\alpha^+ + p)\,\alpha\Sigma_f + (\eta\alpha^+ - 1)\Sigma_t]\,\phi(\boldsymbol{\nu}) = 0, \tag{4}$$

2. Best generation-time approximation:

$$(\psi M^T\phi^+)\,\phi^+(\boldsymbol{\nu}) = 0 = (\psi^+ M\phi)\,\phi(\boldsymbol{\nu}) \tag{5}$$

$$[D_t\nabla^2 - \Sigma_t + \alpha p\Sigma_t]\phi(\boldsymbol{\nu}) = 0$$

$$\left[D_f\nabla^2 - \Sigma_f + \frac{p\Sigma_f}{\alpha^+}\right]\phi^+(\boldsymbol{\nu}) = 0 \quad . \tag{6}$$

In many special cases the two equations of 2 are equivalent (self adjoint) and reduce to Eq. (4), but this is not generally true.

I think the result illustrates the significance of asking the right question of the variational method and, indeed, that few-group-constant computations may be appropriate for only one application and should be recomputed in principle if a different question is to be answered. Other references[4] contain more detail of the practical application of the idea discussed here.

This letter is based on some work presented at the UK Atomic Energy Establishment, Winfrith, May 1969, and I am grateful for their hospitality.

Jeffery Lewins

Mechanical Engineering Department
University College London
Torrington Place
London, W.C.1., England

December 8, 1969

---

[1] E. KIEFHABER, *Nucl. Sci. Eng.*, **38**, 178 (1969).
[2] G. C. POMRANING; *J. Soc. Ind. Appl. Math.*, **13**, 511 (1965).
[3] J. A. LEWINS, *Nucl. Energy* (Nov/Dec 1969).
[4] J. LEWINS, "Developments in Perturbation Theory," *Adv. Nucl. Sci. Technol.*, **5**, 309, Academic Press, New York, N.Y. (1968).