

and show that it has the following generalized importance interpretation: A neutron at the point x in phase space has a probability $\phi^+(x)$ of eventually being detected by the given detector. We can therefore write (f^+, ϕ) as the detector response in the perturbed system since it is the integral of the perturbed flux times the detector cross section. Accordingly, the quantity we want to calculate, the change in the detector response as a result of the perturbation, is given by

$$\begin{aligned} \Delta(f^+, \phi) &= (f^+, \phi) - (f_0^+, \phi_0) \\ &= (f^+, \phi) - (f_0^+, \phi_0) - (f_0^+, \phi) + (f_0^+, \phi) \\ &= (\Delta f^+, \phi) + (f_0^+, \Delta \phi) \\ &= (\Delta f^+, \phi) + (H_0^+ \phi_0^+, \Delta \phi) \end{aligned}$$

where we have added and subtracted the same quantity to go from the first to the second line. The second term in the fourth line can be rewritten as

$$\begin{aligned} (H_0^+ \phi_0^+, \Delta \phi) &= (\phi_0^+, H_0 \phi) - (\phi_0^+, H_0 \phi_0) \\ &= (\phi_0^+, H_0 \phi) - (\phi_0^+, f_0) + (\phi_0^+, f) - (\phi_0^+, f) \\ &= (\phi_0^+, H_0 \phi) + (\phi_0^+, \Delta f) - (\phi_0^+, H \phi) \\ &= (\phi_0^+, \Delta f) - (\phi_0^+, \Delta H \phi) \end{aligned}$$

so that we have the desired result

$$\Delta(f^+, \phi) = (\Delta f^+, \phi) + (\phi_0^+, \Delta f) - (\phi_0^+, \Delta H \phi) \quad (23)$$

A first-order form has been given by Lewins^{5,6} and a more general form, which is both exact and stationary, can be derived.⁷

To obtain Eq. (4) of Sec. I from Eq. (19), we make the following identifications:

$$\begin{aligned} \lambda &= k \\ \phi &= \begin{pmatrix} \phi \\ \mathbf{j} \end{pmatrix} \end{aligned} \quad (24)$$

a $2N$ -component column vector, the first N components of which are the group fluxes, while the last N components are the group currents (which in turn are vectors in three-dimensional physical space).

$$H = \begin{pmatrix} B & 0 \\ 0 & 0 \end{pmatrix}, \quad K = \begin{pmatrix} A & \nabla \cdot \\ -\nabla & -D^{-1} \end{pmatrix} \quad (25)$$

$2N \times 2N$ matrix operators; the element $\nabla \cdot$ is to be interpreted as the divergence operator multiplied by the $N \times N$ identity matrix.

The $2N$ -component adjoint state vector is defined by

$$\phi^+ = \begin{pmatrix} \phi^+ \\ \mathbf{j}^+ \end{pmatrix} \quad (26)$$

and the inner product by

$$(\phi^+, \phi) = \sum_{n=1}^N \int dv (\phi_n^+ \phi_n + \mathbf{j}_n^+ \cdot \mathbf{j}_n) \quad (27)$$

where the space integration is carried out over the entire reactor, and $\mathbf{j}_n^+ \cdot \mathbf{j}_n$ indicates the scalar product of the group n currents in three dimensions. It is then a straightforward matter to verify, using Gauss' theorem, that the adjoint operators are

$$H^+ = \begin{pmatrix} B^+ & 0 \\ 0 & 0 \end{pmatrix}, \quad K^+ = \begin{pmatrix} A^+ & \nabla \cdot \\ -\nabla & -D^{+^{-1}} \end{pmatrix} \quad (28)$$

Writing the direct and adjoint Eqs. (15) in component form, we have

$$\begin{aligned} \nabla \cdot \mathbf{j} + A \phi &= \frac{1}{k} B \phi \\ \mathbf{j} &= -D \nabla \phi \end{aligned} \quad (29)$$

and⁸

$$\begin{aligned} \nabla \cdot \mathbf{j}^+ + A^+ \phi^+ &= \frac{1}{k} B^+ \phi^+ \\ \mathbf{j}^+ &= -D^+ \nabla \phi^+ \end{aligned} \quad (30)$$

It follows that this current-flux representation is equivalent to the usual second-order form [Eq. (1)] of the reactor equations.

To evaluate the perturbation expression [Eq. (19)], we have only to note that

$$\Delta H = \begin{pmatrix} \Delta B & 0 \\ 0 & 0 \end{pmatrix}, \quad \Delta K = \begin{pmatrix} \Delta A & 0 \\ 0 & -\Delta D^{-1} \end{pmatrix} \quad (31)$$

so that

$$\begin{aligned} (\phi_0^+, \Delta H \phi) &= \int dv \tilde{\phi}_0^+ \Delta B \phi \\ (\phi_0^+, \Delta K \phi) &= \int dv (\phi_0^+ \Delta A \phi - \tilde{\mathbf{j}}_0^+ \cdot \Delta D^{-1} \mathbf{j}) \\ (\phi_0^+, H_0 \phi) &= \int dv \phi_0^+ B \phi \end{aligned} \quad (32)$$

The relations of Secs. II and III can be obtained in an analogous manner.

⁸The negative sign in the adjoint current equation is a consequence of the particular choice of the matrix operator K . It has the advantage that the formalism becomes self-adjoint in the limiting one-group case.

Corrigendum

W. M. STACEY, Jr., "Continuous Slowing Down Theory Applied to Fast Reactor Assemblies," *Nucl. Sci. Eng.*, **41**, 381 (1970).

The square brackets in Eq. (2) should contain

$$\left[\frac{\exp(u' - u) - \alpha_i}{1 - \alpha_i} \right] \text{ instead of } \left[\frac{\exp(u' - u)}{1 - \alpha_i} \right]$$

⁵J. LEWINS, *Trans. Am. Nucl. Soc.*, **7**, 211 (1964).
⁶J. LEWINS, "Developments in Perturbation Theory," in *Advances in Nuclear Science and Technology*, P. GREEBLER and E. J. HENLEY, Eds., Vol. 4, p. 309, Academic Press, New York (1968).
⁷D. SELENGUT, *Trans. Am. Nucl. Soc.*, **5**, 413 (1962).