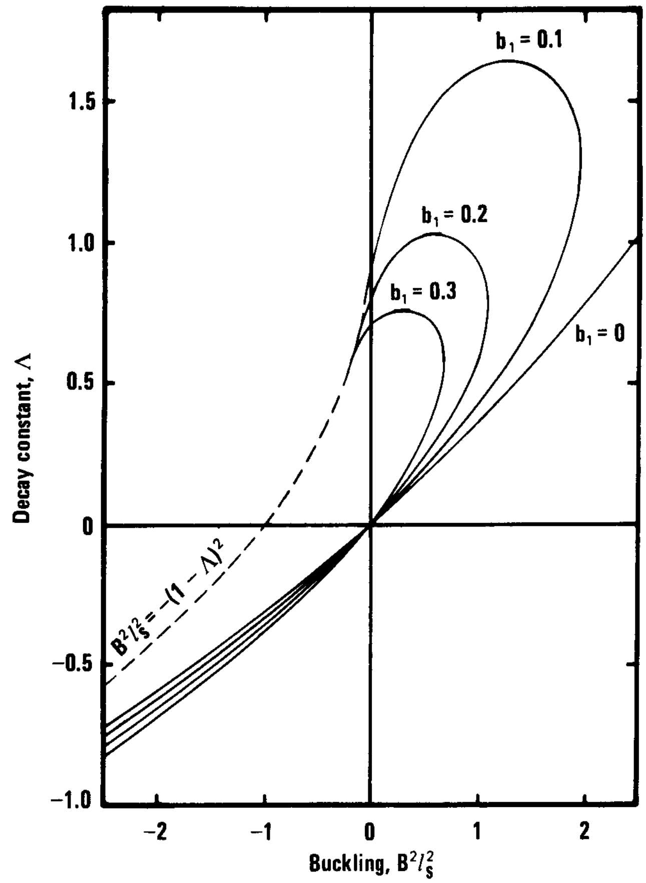

Fig. 1. The generalized decay constant as a function of buckling for various degrees of linearly anisotropic scattering.

Also the more general Figs. 1 and 2 in Ref. 1 agree with the results obtained in Refs. 7 and 8 (apart from the fact that the vertical border lines in Fig. 1 should have been drawn at $1/f_1$ at ±3 instead of at ±4). From Ref. 7 it is clear that region II in Fig. 2 of Ref. 1 frequently contains four imaginary eigenvalues. When they disappear, it is probable that they go over into four complex eigenvalues, just as is the case for purely linear anisotropy. However, this has not been confirmed by calculations.

N. G. Sjöstrand

Department of Reactor Physics
Chalmers University of Technology
S-41296 Göteborg, Sweden

December 11, 1979

Reply to a Comment on "Eigenvalues of the Neutron Transport Equation with Anisotropic Scattering"

The original idea in our paper[1] was to give a general analytical method to determine the number and the mathematical property (real, purely imaginary, or complex) of the discrete eigenvalues of the monoenergetic neutron transport equation with anisotropic scattering. Sjöstrand's comment[2] suggested that his numerical study on the time-dependent transport problem can be compared with our work and his numerical result show agreement with our analytical prediction.

Sjöstrand[2] also discussed the existence of the complex discrete eigenvalues. Just as illustrated in Sec. IV of Ref. 1, the difficulty is how to determine the condition on parameters $c, f_1, f_2, \ldots$ for the existence of complex eigenvalues, where c is the number of the secondary neutrons per collision and $f_1, f_2, \ldots$ are the Legendre coefficients of the scattering function. This problem for the linearly anisotropic scattering case had been considered by Thielheim and Claussen.[3] In Fig. 1 of their paper, they give a boundary of the complex discrete eigenvalues without showing how to determine such a boundary. A very basic property of the quadratically algebraic equation

$$x^2 + 2ax + b = 0 \ , \qquad (1)$$

may help us to solve this problem. This property is that the boundary of the complex roots of Eq. (1), $a^2 - b = 0$, is the condition that Eq. (1) has double roots. With this in mind, we may ask whether this simple property is true for the present problem. That this property is correct for the linearly anisotropic scattering case can be shown as follows.

The discrete eigenvalues for the linearly anisotropic scattering case are roots of the equation

$$\Lambda(\nu^2) = 1 + 3cf_1(1-c)\nu^2 - c[1 + 3f_1(1-c)\nu^2]f(\nu^2) \ ,$$
$$= 0 \ , \qquad (2)$$

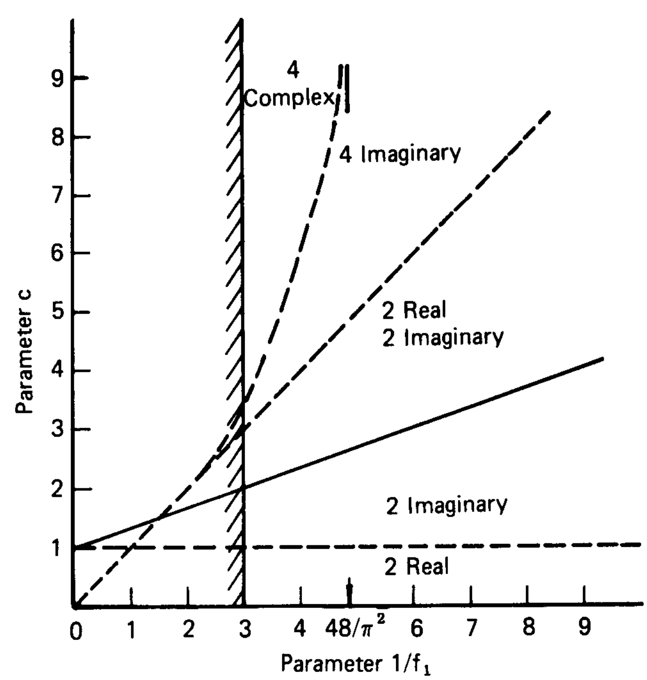

Fig. 1. Classification of discrete eigenvalues in parameter space for linearly anisotropic scattering.

[1] T. DAWN and I. CHEN, *Nucl. Sci. Eng.*, 72, 237 (1979).

[2] N. G. SJÖSTRAND, *Nucl. Sci. Eng.*, 74, 154 (1980).

[3] K. O. THIELHEIM and K. CLAUSSEN, *Kernenergie*, 16, 321 (1973).

where

$$f(\nu^2) = \frac{\nu}{2} \ln \frac{\nu + 1}{\nu - 1} .$$

In addition to Eq. (2), the double roots of Eq. (2) also satisfy $(d/d\nu^2)\Lambda(\nu^2) = 0$ or

$$3f_1(1 - c)[1 - f(\nu^2)] - \frac{1}{2}[1 + 3f_1(1 - c)\nu^2]\left[\frac{f(\nu^2)}{\nu^2} - \frac{1}{\nu^2 - 1}\right] = 0 . \quad (3)$$

If we could eliminate ν^2 from Eqs. (2) and (3), the equation thus obtained would be the boundary we desired. Unfortunately the algebraic manipulations are too tedious to be carried out in detail. A feasible method is to solve c and f_1 from Eqs. (2) and (3). The final result can be expressed as

$$c(\nu^2) = \frac{1 + 3(\nu^2 - 1)[1 - f(\nu^2)]}{1 + (\nu^2 - 1)[1 - f(\nu^2)][1 + 2f(\nu^2)]} , \quad (4a)$$

$$1/f(\nu^2) = \frac{6\nu^2(\nu^2 - 1)[1 - f(\nu^2)]^2}{1 + (\nu^2 - 1)[1 - f(\nu^2)]} c(\nu) , \quad (4b)$$

in which ν^2 is regarded as a parameter varied from $-\infty$ to 0. Equations (4a) and (4b) are a parametric representation of the boundary of the complex discrete eigenvalues in the parameter (c, $1/f_1$)-space (shown in Fig. 1). From Eqs. (4a) and (4b), it can be easily proved that $c \to \infty$ and $f_1 \to \pi^2/48$ as $\nu^2 \to -0$, which is discussed by many authors.[3,4] The present result is compatible with those numerical results developed by Thielheim and Claussen[3] as well as Sjöstrand.[4] (See, in particular, Fig. 2 in Ref. 3 and Fig. 1 in Ref. 4.)

Perhaps a similar technique can be used in studying the case of quadratically anisotropic scattering as well. We cannot have recourse to machine computation due to the prohibitively large amount of manipulations. The computation is still in progress.

The vertical border lines of Fig. 1 in Ref. 1, mentioned by Sjöstrand,[2] may need some explanation. In preparing this figure, $f_2 = -1/20$ is taken as an example. With this particular value, the condition for a positive scattering function [Eq. (7) in Ref. 5] becomes $-\frac{1}{4} \leqslant f_1 \leqslant \frac{1}{4}$, which is shown correctly as border lines in the figure.

Trine-Yie Dawn

Department of Nuclear Engineering
National Tsing Hua University
Hsin-chu, Taiwan, Republic of China

January 16, 1980

[4]N. G. SJÖSTRAND, "Decay Constants for Pulsed Monoenergetic Neutron Systems with Anisotropic Scattering," CTH-RF-28, Chalmers University of Technology (1975).

[5]E. A. ATTIA, A. A. HARMS, and S. A. KUSHNERIUK, *Can. J. Phys.*, **53**, 825 (1975).