$$l = \frac{1}{\text{removal rate}}$$

and

$$\Lambda = \frac{1}{\text{production rate}}$$

and define

$$k_{\text{eff}} = \frac{\text{production rate}}{\text{removal rate}}$$

and

$$\rho = \frac{\text{production rate} - \text{removal rate}}{\text{production rate}} .$$

Then we have

$$\rho = \left(\frac{1}{\Lambda} - \frac{1}{l}\right)\Big/\frac{1}{\Lambda} = (l - \Lambda)/l .$$

In a *critical* reactor, $\rho = 0$ and $l = \Lambda$ or $l = \tau/\nu$.

The maximum reactivity possible (which relates perhaps to Marotta's concept of efficient utilization) will come when all neutron losses are zero. Putting $\Sigma_a = \Sigma_f + \Sigma_c$, "losses" are due to $\Sigma_c + DB^2$:

$$\rho = \frac{(\nu - 1)\Sigma_f - (\Sigma_c + DB^2)}{\nu\Sigma_f} \; ; \quad \rho_{\max} \to \frac{\nu - 1}{\nu} .$$

Correspondingly,

$$\Lambda = \frac{1}{\nu\Sigma_f v} \; ; \quad l_{\max} \to \frac{1}{\Sigma_f v} \; ; \quad \tau_{\max} \to \frac{1}{\Sigma_f v} .$$

Thus, $l - \tau \to 0$ at the physical maximum and $l - \Lambda \to l(\nu - 1)/\nu$. In the Monte Carlo sense, $\tau$ but not $\Lambda$ is a subset of $l$.

It is not clear to me whether Marotta's $g$ is my $\tau$ or my $\Lambda$. I would also comment that the choice of weighting function in Monte Carlo calculations, like other calculations of static eigenvalues, introduces a somewhat arbitrary linear scaling of $\rho$ (or $k_{\text{eff}}$) (Ref. 6) when the properties are not uniform in space and velocity as supposed in the simple model used here.

I hope the above simple account of the concepts helps clarify misunderstandings to which I may have contributed.

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December 23, 1980

[6]A. M. WEINBERG and E. P. WIGNER, *The Physical Theory of Neutron Chain Reactors*, Chicago (1958).

## Response to "Neutron Lifetime, Generation Time, and Reproduction Time"

I thank Lewins for clarifying in Ref. 1 some ambiguous basic ideas and nomenclature in this area. I would like to compare the definitions of $g$ and $l$ as I used them in Ref. 2 and their relationship to $\Lambda$ and $\tau$ as used by Lewins in Ref. 1. I follow Lewins' format in the comparison Table I.

The KENO generation time $g$ is calculated by multiplying the neutron elapsed time by the $\nu$-fission probability and thus is equivalent to Lewins' $\Lambda$. Unfortunately, different names are used for the same concept. Lewins' suggested reproduction time should be adopted—especially since generation time (as introduced by Hurwitz) has historic precedence.

I agree with Lewins' formulation in Ref. 1 that in a critical reactor one should expect $l$ and $g$ (= $\Lambda$) to be equal. I also agree that, at the physical maximum state of $k_{\text{eff}}$, $l$ should equal $\bar{\nu}g$ (= $\tau$) where $\bar{\nu}$ is the average number of fast neutrons produced per fission.

The above equalities are at variance with my results of Ref. 2. There it was established, for a complex coupled fissionable system, that $l$ and $g$ (= $\Lambda$) are equal only at the point of maximum utilization of neutrons (excess time $E = l$ and $g = 0$), which happened to be also the point of maximum $k_{\text{eff}}$ of the configuration. The maximum slope of $k_{\text{eff}}$ indicates the point of maximum neutron utilization ($E = 0$, $l = g$) of the system. In this calculation, it was also shown that two critical states exist for the system; both, however, indicate that $l$ and $g$ are not equal for $k_{\text{eff}}$ of unity. It would therefore appear that the elementary theory of Ref. 1 cannot account for complex interaction and therefore cannot be used for reliable guidance for reactivity values or trends.

Two uncoupled simple systems have been analyzed using the same methodology as in Ref. 2 to explore further the $l$ and $g$ relationship. I calculated $k_{\text{eff}}$ versus moderator (water) density for one of the 200 fuel assemblies comprising the array of Ref. 2. This was a 17 X 17 U(3)$O_2$-rod[3] light water reactor fuel assembly submerged in water with a 1-ft-thick all-around water reflector. The pertinent KENO results are given in Table II and Fig. 1, where $k_{\text{eff}}$ and $E$ have been added as dashed curves to Fig. 1 of Ref. 2. We note that at full moderator density, $k_{\text{eff}}$ of the assembly agrees well with that of the array since all the 200 assemblies of the 20 X 10 array

TABLE I

| Parameter | Designation by | |
|---|---|---|
| | KENO[a] | Lewins[b] |
| Neutron lifetime[c] | $l$ | $l$ |
| Neutron reproduction time[d] | $g$ (called generation time in KENO) | $\Lambda$ |
| Generation time[e] (Hurwitz) | Not used | $\tau$ |

[a]As used in Ref. 2.
[b]As used in Ref. 1.
[c]The average life span of a neutron until it escapes or gets absorbed.
[d]The average time it takes a neutron to produce another neutron.
[e]The average time taken by a neutron to cause a fission in a steady-state fission distribution. See Ref. 4.

[1]J. D. LEWINS, *Nucl. Sci. Eng.*, **78**, 105 (1981).
[2]C. R. MAROTTA, *Nucl. Sci. Eng.*, **77**, 107 (1981).
[3]Here U(3) denotes uranium enriched to 3% in the $^{235}$U isotope.
[4]H. HURWITZ, Jr., *Nucleonics*, **5**, 61 (July 1949).

TABLE II

KENO* Results for Single Assemblies

| Moderator Density ($g/cm^3$) | 17 X 17 Rod Assembly | | | | 30 X 30 Rod Assembly | | | |
|---|---|---|---|---|---|---|---|---|
| | $k_{eff}$ | Neutron Lifetime,[a] $l$ ($10^{-4}$ s) | Generation Time,[b] $g$ ($10^{-4}$ s) | Excess Time, $E$ $(l-g)$ ($10^{-4}$ s) | $k_{eff}$ | Neutron Lifetime,[a] $l$ ($10^{-4}$ s) | Generation Time,[b] $g$ ($10^{-4}$ s) | Excess Time, $E$ $(l-g)$ ($10^{-4}$ s) |
| 0 | 0.083 | --- | --- | --- | 0.128 | --- | --- | --- |
| 0.05 | 0.097 | 0.072 | 0.998 | -0.926 | 0.176 | 0.110 | 0.660 | -0.550 |
| 0.10 | 0.173 | 0.672 | 1.578 | -0.906 | 0.295 | 0.741 | 1.278 | -0.537 |
| 0.15 | 0.274 | 1.697 | 2.346 | -0.649 | 0.417 | 1.581 | 1.674 | -0.093 |
| 0.20 | 0.350 | 2.535 | 2.319 | 0.216 | 0.514 | 2.172 | 1.621 | 0.551 |
| 0.30 | 0.451 | 3.256 | 1.731 | 1.525 | 0.658 | 2.640 | 1.232 | 1.408 |
| 0.45 | 0.553 | 3.027 | 1.133 | 1.894 | 0.800 | 2.133 | 0.765 | 1.368 |
| 0.60 | 0.664 | 2.377 | 0.799 | 1.578 | 0.903 | 1.626 | 0.552 | 1.074 |
| 0.80 | 0.751 | 1.751 | 0.576 | 1.175 | 1.000 | 1.144 | 0.420 | 0.724 |
| 1.00 | 0.839 | 1.295 | 0.459 | 0.836 | 1.091 | 0.846 | 0.360 | 0.486 |

*All $k_{eff}$, $l$, and $g$ calculated using the 123 GAM-THERMOS cross-section set and 15 000 neutron histories; $k_{eff}$ values reported are within ±0.004 for one standard deviation; $l$ and $g$ are within $\pm 1.5 \times 10^{-6}$ s for one standard deviation.

[a]Average life span of a neutron in the system until it escapes from the system or is absorbed.

[b]Average time it takes a neutron to produce another neutron.

are neutronically isolated at full density water. The value of $k_{eff}$ falls off monotonically as the water density is reduced from 1.0 $g/cm^3$ to zero. Here, $E$ is zero ($l = g$) between $\rho$(MOD) of 0.15 and 0.20, exactly in the region in which the slope of $k_{eff}$ is maximum. In the moderator density range from 0 to 0.2 $g/cm^3$, the fast fissions in $^{238}U$ are contributing more efficiently (in a rather futile way) toward a chain reaction than is the flattened $k_{eff}$ region from 0.2 to 1.0 $g/cm^3$.

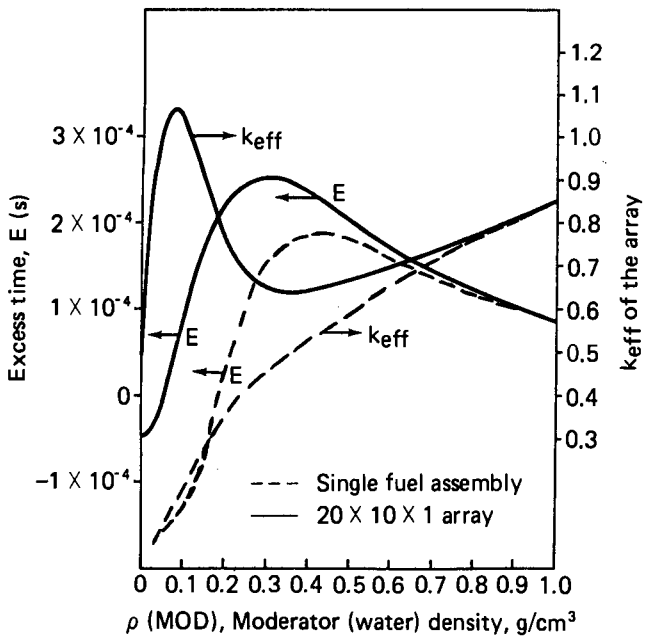


Fig. 1. The values of $k_{eff}$ and $E$ as a function of moderator density for the finite reflected (20 X 10 X 1) array and for a single fuel assembly.

The second system was a simple extension of the above 17 X 17 lattice to a 30 X 30 rod lattice at the same pitch. This arbitrary enlargement of the fuel assembly was made to effect $k_{eff}$'s of unity and greater. This new assembly was also submerged in water with a 1-ft-thick all-around water reflector and $k_{eff}$ was calculated with the same moderator perturbation as in the previous examples. The results are given in Table II; we note that at critical, $l$ and $g$ are not equal. The value of $E$ is equal to zero ($l = g$) at approximately the same point as for the 17 X 17 case, demonstrating also the same sharp increase in $k_{eff}$ (maximum utilization) due to fast fissions in $^{238}U$.

It appears from the examples given, using the concept of excess time $E$, defined through $l$ and $g$, and using the definitions of $l$ and $g$ as stated in Table I, that $l$ is equal to $g$ ($= \Lambda$) only at the time of maximum neutron utilization ($E = 0$) as a chain reacting process is being established and not necessarily at critical. This is true both for simple uncoupled homogeneous systems as well as complex coupled ones. The point of maximum $k_{eff}$ would in most cases for complex coupled systems coincide with the point of maximum neutron utilization. It also appears that there is no relationship in general at critical between $l$ and $g$ for complex or simple systems, coupled or uncoupled.

My KENO Monte Carlo calculations reported in Ref. 2 and in this Letter did not use any adjoint weighting or any other importance sampling device to bias the neutron histories being tallied for the $k_{eff}$ calculations. The Monte Carlo game was played fair using a sufficient number of histories with adequate sampling from all regions of space to insure an unbiased convergence to the "proper" $k_{eff}$.

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January 19, 1981