

Letters to the Editor

On "Double Criticality"

Ishiguro et al.¹ claim to have been the first to observe and define "double criticality," which they demonstrate in a uniform-power-reflected reactor. In connection with this statement, I would like to report a similar phenomenon occurring in heavy-water reactors.²

Consider a heavy-water-moderated and -reflected slab reactor fueled with uniformly distributed natural uranium and inquire how the local burnup of the fuel should be distributed so that the average burnup will be maximal—one finds that

the local burnup distribution follows the curve in Fig. 1 (Ref. 3). The corresponding neutron flux distribution is then given by the formula

$$[\phi(x)/\phi(0)]^2 = \gamma + (1 - \gamma) \cos^2 \bar{\kappa} x, \quad (1)$$

where γ is a constant given by the character of the dependence of the buckling $\bar{\kappa}$ on the burnup s (here taken as a quadratic). From the boundary and continuity conditions on the core-reflector interface h , follows the criticality condition:

$$\gamma = -[\tan \bar{\kappa} h \tan(\delta + \bar{\kappa} h)]^{-1}, \quad \delta = \tan^{-1}(\bar{\kappa} \tanh t), \quad (2)$$

¹Y. ISHIGURO et al., *Trans. Am. Nucl. Soc.*, **33**, 782 (1979).

²V. BARTOŠEK, *Czech. J. Phys.*, **B21**, 1310 (1971).

³V. BARTOŠEK, *Industries Atomiques*, 11-12, 23 (1970).

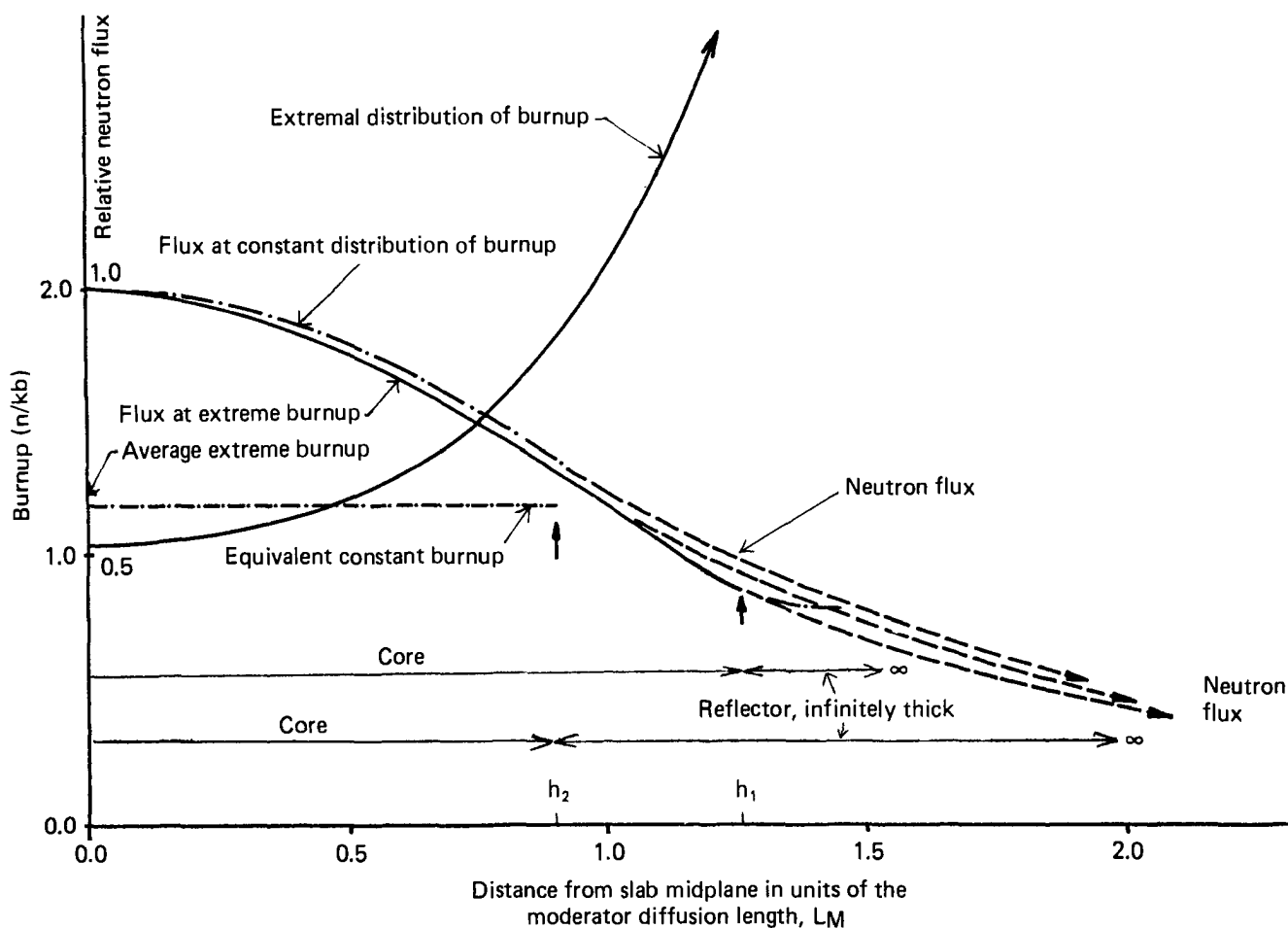


Fig. 1. Burnup distribution of natural uranium in a heavy-water reactor giving two critical values of half-thickness h_1 and h_2 .

which gives two values for the core half-thickness, $h_1 > h_2$, if the reflector half-thickness t is given ($0 < t \leq \infty$, with distances measured in units of moderator diffusion length L_M). This means that for identical burnup distributions, two half-thicknesses of the reactor core, i.e., "double criticality," occur.

We first consider the physical interpretation of the phenomenon with the case of infinite reflector. On the core-reflector interface, the burnup attains the value $s(h_2)$. Extending the core into the reflector by uniformly adding fuel with burnup following the prescribed distribution into the region $(h_1 - h_2)$, two trends compete: the increase of reactivity due to the larger core thickness and its decrease due to the increasing burnup of the added fuel. At the beginning of the procedure the former effect prevails, but finally the second becomes more effective. The two effects cancel at core half-thickness h_1 , making the larger core critical. It is evident that a region with negative buckling will accrue in the core.

It can be perhaps claimed that in the case described, a part (with half-thickness h_2) of a critical reactor (with half-thickness h_1) is also critical.

In the reactor with a finite reflector two cases could be distinguished as follows.

1. Extracting the fuel and moderator from the region $h_1 - h_2$ and shifting the reflector to the core, the resulting smaller reactor will also maximize the average burnup and h_2 will be given by the root of Eq. (2).

2. Extracting only fuel from the region $h_1 - h_2$ also changes the value of h_2 because this also affects the reflector thickness, changing it to the value $h_1 - h_2 + t$. In this case, a part of the larger reactor will be critical as well, but the (unchanged) burnup distribution in the core will not yield the maximal average burnup of the fuel.

Consequently it seems to be certain that in a reactor with nonuniform distribution of physical parameters (multiplication, absorption, etc.) a "strange" behavior (e.g., double criticality) may occur, probably due to the fact that the region with negative buckling is present in the core.

I wonder whether the double criticality described in Ref. 1 has any physical explanation, or whether it is merely the consequence of the mathematical model used; namely, that of two-group theory.

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Reply to "On 'Double Criticality' "

During the preparation of our earlier communication,¹ we were surprised there were no references to "double criticality" and apologize to Bartosek for not having been aware of his publications.

In response to his Letter,² we point out that the problems discussed in Refs. 1 and 2 are slightly different.

If a symmetric slab reactor is parted at the center and if the space is filled with a material whose k_∞ is unity, the new system is critical for any size of the central region. In our

problem, the imposed conditions, such as the continuity of the fuel density, happen to be met at one particular point.

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Editor's Note: Although the above exchange stems from a communication not in *Nuclear Science and Engineering*, thereby not meeting one of the usual criteria of Letters, we believe the Society has a responsibility to provide a forum for comment no matter which Society publication is involved. Until more appropriate outlets develop, *Nuclear Science and Engineering* will consider providing, upon request, such a forum.

Neutron Lifetime, Generation Time, and Reproduction Time

Marotta has courteously shown me an advance copy of his work employing the concept of the excess time.¹ I am no Monte Carlo expert and cannot usefully comment on the application in this area. But I feel I am responsible for a certain amount of confusion in giving the name "generation time" some years² ago to a concept pioneered by Henry.³ May I make belated amends? I would now prefer the name "reproduction time"⁴ and distinguish this from what Hurwitz⁵ called the generation time. I hope the following explanation with values in the simplest model of a reactor will make the distinction:

l = neutron lifetime

= mean time for one neutron to be removed from the reactor

$$= \frac{1}{(\Sigma_a + DB^2)v}$$

Λ = neutron reproduction time

= mean time for one neutron to be replaced by another neutron on fissioning

$$= \frac{1}{v\Sigma_f v}$$

τ = neutron generation time

= mean time for one neutron to cause fission, i.e., to bring about the next generation

$$= \frac{1}{\Sigma_f v}$$

It is well known⁴ that what is "production" and what is "removal" [e.g., $(n, 2n)$ processes] is something of an arbitrary definition, but within such limits, one can say

¹C. R. MAROTTA, *Nucl. Sci. Eng.*, **77**, 107 (1981).

²J. LEWINS, *Nucl. Sci. Eng.*, **7**, 122 (1960).

³A. F. HENRY, *Nucl. Sci. Eng.*, **3**, 52 (1958).

⁴J. LEWINS, *Nuclear Reactor Kinetics and Control*, Pergamon Press Ltd., Oxford, United Kingdom (1978).

⁵H. HURWITZ, Jr., *Nucleonics*, **5**, 61 (July 1949).

¹Y. ISHIGURO et al., *Trans. Am. Nucl. Soc.*, **33**, 372 (1979).

²VACLAV BARTOSEK, *Nucl. Sci. Eng.*, **78**, 104 (1981).