Letters to the Editor

On Taylor Series Expansions with Generalized Perturbation Methods

A recent paper by Gandini[1] concerning generalized perturbation theory (GPT) for nonlinear systems contains a discussion of GPT application to functionals involving derivative operators (see footnote 26 of Ref. 1). Gandini[1] gives as an example of this application a Taylor series expansion of a perturbed function in time, which is basically the same approach he suggested to us[2] to investigate spatial shift effects in our studies of peak power density sensitivities and uncertainties for a heterogeneous liquid-metal fast breeder reactor (LMFBR) core.[3] The purpose of this Letter is to demonstrate the ease of application of this approach with existing GPT codes, and to verify the appropriateness of this method for a certain class of problems.

The parameter that we investigated[3] is the ratio of the point power density at $\boldsymbol{r}$ to the total power (PD):

$$PD(\boldsymbol{r}) = \frac{\sum_j \Sigma_{1j}(\boldsymbol{r})\phi_j(\boldsymbol{r})}{\int_{\text{reactor}} \sum_j \Sigma_{2j}(\boldsymbol{r}')\phi_j(\boldsymbol{r}')d\boldsymbol{r}'} , \tag{1}$$

where the Σ_{ij} are the energy production cross sections for group j defined with certain simplifying assumptions.[3] Sensitivities, $\delta PD/\delta\sigma$, of PD to various nuclear data (σ) were determined with GPT methods.

In Ref. 3 we considered the influence on these sensitivities of perturbation-caused "far-range" shifts of the maximum PD, e.g., between various driver zones, and "near-range" shifts near the location of the initial maximum PD in a particular driver zone. Furthermore, we utilized an r-z model and did not consider strongly localized perturbations, e.g., those for ^{10}B. It is for these "near-range" effects that Gandini proposed the Taylor series expansion model.[2]

For the further discussion, we consider a simplified geometry (one-dimensional slab) and response function for which the Σ_{ij} in Eq. (1) are all assumed to be 1.0. Thus the parameter of interest is

$$R(x) = \frac{\sum_j \phi_j(x)}{\int_{\text{reactor}} \sum_j \phi_j(x')dx'} = \frac{\Phi(x)}{\int_{\text{reactor}} \Phi(x')dx'} . \tag{2}$$

The basic method proposed by Gandini to determine the perturbed value, $R'(x)$, of R at various distances from the point x_0 involves the following Taylor expansion:

$$R'(x) = R'(x_0) + R^{(1)'}(x_0)(x - x_0) + \frac{1}{2!}R^{(2)'}(x_0)(x - x_0)^2 + \dots , \tag{3}$$

where the perturbed values of the i'th derivative, $R^{(i)'}$, are determined[1,2] through an extended form of GPT. The following discussion will be restricted to the truncated three-term expansion of Eq. (3).

An attractive feature of Eq. (3) is that it is simple to apply to the numerical technique generally used in direct flux and GPT calculations, i.e., the finite difference method. Thus $R^{(1)'}$ and $R^{(2)'}$ at x_k can be approximated using functions at x_{k-1}, x_k, and x_{k+1}. Hence, the programming to generate the necessary generalized adjoint functions (Γ^*) to determine the $R^{(i)'}$ with GPT can be quite simply incorporated into existing GPT codes, which calculate R', such as the VENTURE/DEPTH-CHARGE chain,[4,5] the FORSS system,[6] and the Italian GPT package.[7-9] As pointed out by Williams[10] for the case of depletion perturbation theory, perturbation expressions should be based on the approximation equations used for the direct solution of the problem.

In fact, with some very modest processing of the output of existing GPT codes, the desired functions can be determined with no further programming of these codes. One can determine the desired $R^{(i)'}(x_k)$ by using the R' at x_k and its nearest neighboring mesh points directly. Alternatively, since the desired Γ^* are solutions to fixed source problems, one can take advantage of the cumulative nature of such solutions to determine the Γ^* functions necessary to calculate $R^{(i)'}$ from an appropriate sum of the generalized adjoint functions used to obtain R'.

Some brief calculations were performed to test the validity

[1] A. GANDINI, *Nucl. Sci. Eng.*, **77**, 316 (1981).

[2] A. GANDINI, Private Communication (July 29, 1980).

[3] J. M. KALLFELZ et al., "Design and Sensitivity Analysis of a CDS-Type Heterogeneous Core," *Proc. Topl. Mtg. Advances in Reactor Physics and Shielding*, Sun Valley, Idaho, September 14-17, 1980, p. 467, American Nuclear Society (1980).

[4] D. R. VONDY et al., "VENTURE: A Code Block for Solving Multigroup Neutronics Problems Applying the Finite-Difference Diffusion Theory Approximation to Neutron Transport, Version II," ORNL-5062/R1, Oak Ridge National Laboratory (1977).

[5] J. R. WHITE, "The Development, Implementation, and Verification of Multicycle Depletion Perturbation Theory for Reactor Burnup Analysis," ORNL/TM-7305, Oak Ridge National Laboratory (1980).

[6] J. L. LUCIUS et al., "A User's Manual for the FORSS Sensitivity and Uncertainty Analysis Code System," ORNL-5316, Oak Ridge National Laboratory (1980).

[7] I. DAL BONO, "TAIM-Multigroup Diffusion Code," CEC(66)12, Comitato Nazionale per l'Energia Nucleare (1966).

[8] I. DAL BONO et al., "The CIAP-1D Code," RT/FI(68)9, Comitato Nazionale per l'Energia Nucleare (1968).

[9] I. DAL BONO et al., "The GLOBPERT-1D Code," RT/FI(68)10, Comitato Nazionale per l'Energia Nucleare (1968).

[10] M. L. WILLIAMS, *Nucl. Sci. Eng.*, **70**, 20 (1979).

of the approach suggested by Gandini,[1,2] utilizing the Italian GPT chain.[7-9] For investigations of the "near-range" effects described above, the x value of interest is generally very near the location of the zonal peak power density.[3] However, to test the proposed method, we chose points in our simple model where both the derivatives $R^{(1)}$ and $R^{(2)}$, and the perturbations thereof, $\delta R^{(1)}$ and $\delta R^{(2)}$, were expected to be appreciable. Since GPT calculates $\delta R^{(i)}$, to avoid loss of significant figures for small perturbations we evaluated the following expression, rather than using Eq. (3) directly:

$$\delta R(x) = \delta R(x_0) + \delta R^{(1)}(x_0)(x - x_0) + \frac{1}{2}\,\delta R^{(2)}(x_0)(x - x_0)^2 \ . \quad (4)$$

For our test calculations, we considered a two-region, one-dimensional slab with sodium, ^{238}U, and ^{239}Pu core and blanket concentrations characteristic of an LMFBR, e.g., those of the test model of Ref. 11. The core and blanket thicknesses were 80 and 30 cm, respectively, and the three-group core cross-section set of the CITATION test case[12] was employed.

A sample perturbation that we considered was a 10% decrease in the core ^{239}Pu density (N_c^{49}) and an addition of ^{239}Pu with a number density of 0.15 N_c^{49} to the blanket. The space range of interest was approximately the middle half of the core, because of the eventual interest in peak power density investigations. While our goal was not to address the agreement between $\delta R_D(x)$ determined by direct calculations and $\delta R_P(x)$ from normal GPT, the ratio of these two values for the above perturbation did not deviate from 1.0 by more than ~10% in the space range of interest. This agreement is obviously a function of the perturbation magnitude, and was within 2% for smaller test perturbations.

To examine the appropriateness of the Taylor series expansion used in GPT, for the above sample perturbation we compared $\delta R_P(x)$ determined by normal GPT at x with $\delta R_{PT}(x)$ resulting from Eq. (4), with all $\delta R^{(i)}(x_0)$ determined by GPT.

We considered several cases in the space range of interest, with $(x - x_0)$ values of 6 cm and $\delta R(x_0)$ and $\delta R(x)$ values that differed by almost a factor of 2. For these cases, the ratio $\delta R_{PT}(x)/\delta R_P(x)$ did not differ from 1.0 by more than ~10%. (For regions near the blanket the agreement was poorer, but these regions are not of general interest for investigations of maximum power densities.)

[11]J. M. KALLFELZ et al., *Nucl. Sci. Eng.*, **62**, 304 (1977).

[12]T. B. FOWLER et al., "Nuclear Reactor Analysis Code: CITATION," ORNL-TM-2496, Rev. 2, Oak Ridge National Laboratory (1971).

Thus the Taylor series expansion method suggested by Gandini[1,2] for GPT appears promising for GPT investigations of point power density sensitivities. For a spatial scan of power density sensitivities, this method can potentially reduce considerably the number of necessary Γ^* calculations. Investigation of this method for various applications and perturbations is continuing.

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Response to "On the Taylor Series Expansion with Generalized Perturbation Methods"

Although preliminary, the results obtained by Perone et al.[1] seem quite encouraging. They again indicate the potentiality of the generalized perturbation theory (GPT) methods in the solution of crucial problems in reactor safety and project domains, apart from the significant insight into complex mechanisms regulating the neutron economy of multiplying systems, which is gained by their use. The quantity specifically analyzed by the authors is the power factor at different (in particular at peak power) reactor positions. There is no doubt that a full understanding of the dependence of such a quantity on basic data (and their inaccuracies), or project parameters, will be highly helpful, either in defining with confidence the operational margins of a power system, or in optimizing its performance in terms of both maximal overall power level and average fuel burnup at the end of cycle.

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[1]VINCENT A. PERONE, JOHN M. KALLFELZ, and LOTFI A. BELBLIDIA, *Nucl. Sci. Eng.*, **79**, 326 (1981).