NUCLEAR SCIENCE AND ENGINEERING: **20**, 111-114 (1964)

Letters to the Editors

A Theorem Concerning the Slowing-Down Equation

The number of collisions required to thermalize neutrons by elastic collisions in a non-absorbing medium has been obtained by DeMarcus (1959)[1]. This derivation is based on a source distribution which will give the asymptotic value of the collision density for all lethargies in the special case of isotropic neutron scattering. The derivation of the source function for small values of the lethargy, as used by DeMarcus, is given in Volume III of the *Reactor Handbook* (1962)[2]. The following theorem proves a relationship between the source and any scattering kernel for the collision density to be constant for all lethargies.

Theorem: Let $F(u)$, $S(u)$ and $K(u)$ be single-valued functions of u and let ϵ be a positive constant. The integral equation:

$$F(u) = \begin{cases} S(u) + \int_0^u K(u-u')F(u')du' & 0 \leq u \leq \epsilon \\ S(u) + \int_{u-\epsilon}^u K(u-u')F(u')du' & u \geq \epsilon \end{cases} \quad (1)$$

defined for all $u \geq 0$, with $\int_0^\epsilon K(u)du = 1$, has a solution which is a constant C for all u, if and only if $S(u)$ and $K(u)$ are connected by the relationship

$$S(u) = \begin{cases} C\,(1 - \int_0^u K(u')du') & 0 \leq u \leq \epsilon \\ 0 & u \geq \epsilon \end{cases} \quad . \quad (2)$$

Proof: Applying the Laplace Transform to (1) and using the convolution theorem leads to:

$$F(u) = \frac{1}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty} e^{pu}\,\frac{\bar{S}(p)}{1 - \bar{K}(p)}\,dp, \quad (3)$$

where $\bar{S}(p)$ and $\bar{K}(p)$ are the Laplace transforms of $S(u)$ and $K(u)$.

The Laplace transform of $S(u)$ can be found from (2), and is

$$\bar{S}(p) = \frac{C}{p}\,\{1 - \bar{K}(p)\}.$$

Substitution for $\bar{S}(p)$ in (3) then gives:

$$F(u) = C \quad \text{for all} \quad u \geq 0.$$

Conversely, if it is assumed that $F(u) = C$, then (1) gives:

$$C = \begin{cases} S(u) + C\int_0^u K(u')\,du' & 0 \leq u \leq \epsilon \\ S(u) + C\int_0^\epsilon K(u')\,du' & u \geq \epsilon \end{cases}$$

or

$$S(u) \begin{cases} C(1 - \int_0^u K(u')du') & 0 \leq u \leq \epsilon \\ 0 & u \geq \epsilon \end{cases} .$$

For isotropic scattering in a single nuclear species, the kernel $K(u)$ is given by:

$$K(u) = e^{-u}/(1 - \alpha) \qquad 0 \leq u \leq \epsilon$$

where α is related to the mass number of the nuclear species and $\epsilon = \ln 1/\alpha$ is the maximum lethargy gain per collision.

The solution of the slowing-down equation will then be a constant, C, if

$$S(u) = \begin{cases} C\left[1 - \frac{1}{1-\alpha}(1 - e^{-u})\right] & 0 \leq u \leq \epsilon \\ 0 & u \geq \epsilon \end{cases} . \quad (4)$$

For unit source $\int_0^\infty S(u)du = 1$, giving $C = 1/\xi$, where ξ, the first moment of the kernel, is equal to $1 + \alpha \ln \alpha/(1 - \alpha)$.

The source distribution given in (4) is that used by DeMarcus.

M. W. Dyos

Australian Atomic Energy Commission
Private Mail Bag
Sutherland, N. S. W.
Australia

Received March 10, 1964

[1] W. C. DeMARCUS, *Nucl. Sci. Eng.* **5**, 336-337 (1959).
[2] H. SOODAK (Ed.), Vol. III, Part A, *Reactor Handbook*, 2nd Ed., p. 125. Interscience Publishers, New York, London, (1962).