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Letters to the Editors

Comment on the Effect of Resonance Correction to Group Flux in Fast-Reactor Doppler-Effect Calculation

This letter expresses what we believe to be a significant disagreement with the point of view of Hummel and Hwang[1,2] regarding the importance of using flux correction factors and making resonance overlap corrections for the overlap between resonances in two different materials.

The paper by Hwang[2] leads one to believe that, for good accuracy, it is necessary to include the resonances of other materials of a mixture when calculating the Doppler effect on reactivity for a particular component of the mixture. In fact, the paper leaves the impression that the effect is quite large. The concluding paragraph of the note by Hummel and Hwang[1] seems to contradict this to some extent, but we find it difficult to be sure what they mean in that paragraph. The note[1], with certain qualifications, also states that it makes little difference whether one considers the so-called 'flux correction factor' when making Doppler-effect calculations.

We believe that it has been shown by Fischer[3] that it is much better to always include the flux correction factor and that Fischer[3], Rowlands[4], Hutchins and Greebler[5], and Froelich and Ott[6] have all shown by slightly different techniques that it is usually unnecessary to consider the effect of overlap between the resonances of two different materials in the mixture.

To avoid defining the complex notation involved in this discussion we adopt the notation of Hummel and Hwang, and reference to Eq. (1.3), for example, will mean Eq. (3) of Ref. 1 and, again for example, Eq. (2.5) will mean Eq. (5) of Ref. 2.

Equations (1.1) define the usual effective cross sections $\tilde{\Sigma}_x$ in the narrow resonance approximation, and Eqs. (1.2) define modified effective cross sections $\tilde{\Sigma}_x^{(e)}$, which differ from $\tilde{\Sigma}_x$ by the flux correction factor f defined by Eq. (1.3). When we say that the flux correction factor is included in the calculation, we mean that the effective cross sections of Eqs. (1.1) are used, and, when it is ignored, the cross sections of Eqs. (1.2) are used. We think that this coincides with the terminology of Hummel and Hwang.

To further clarify the terminology we will define the Doppler temperature coefficient of reactivity for one component of a mixture to be associated with reactivity changes that would be produced when the temperature of that component is changed and the temperature of the other components held constant. We believe that this is the only reasonable and useful definition for the contribution of each component to the Doppler effect. Normally, in a power reactor, the isotopes are fairly homogeneously mixed, so that it is impossible to heat one without the other, but in experimental fast critical assemblies the fissionable and fertile isotope mixtures are usually assembled from thin plates of the individual components, and then in an accidental power excursion the fissionable material would heat considerably more than the fertile material. In the former case there is no loss in defining the coefficient in this way, and in the latter case it is a practical necessity.

Fischer has shown[3] that for typical large fast breeders a change in temperature of the fissionable isotope has negligible effect on the effective cross section of the fertile isotope and vice-versa, when the effective cross sections are defined by Eqs. (1.1), whereas this is not true when the effective cross sections are defined by Eqs. (1.2). If the former is true, the latter follows directly from the fact that f depends upon the temperature of both materials. We believe that this is a very strong point in favor of doing all calculations with the normal effective cross sections as given by Eqs. (1.1) and not bothering to introduce the modified cross sections of Eq. (1.2), which only tend to confuse the problem. The lack of dependence of the cross sections of one isotope upon the temperature of the other means that one can calculate the effective cross section of the fissionable material, ignoring the existence of resonances in the fertile material. But if the modified cross sections are used, one must correctly account for overlap between fertile and fissile resonances, which complicates the calculation as evidenced by Ref. 2. Furthermore, as Hwang points out, the use of the modified cross sections amounts to a reassignment of the total δk to the various isotopes of the mixture. The result of this reassignment is that the individual contributions to δk can no longer be interpreted as being associated physically with the particular isotopes.

The only reason for introducing the modified cross sections is that, if the procedure of Hummel and Hwang actually gives a good approximation to the total Doppler effect of a homogeneous mixture (which we suspect is true but do not believe has been clearly demonstrated as yet), then for this case of a homogeneous mixture it is necessary to calculate only the temperature dependence of $\left\langle \frac{\Sigma_f}{\Sigma_t} \right\rangle$ and $\left\langle \frac{\Sigma_\gamma}{\Sigma_t} \right\rangle$ and not $\left\langle \frac{1}{\Sigma_t} \right\rangle$. This is a very slight advantage considering that one buys the necessity of making overlap corrections. It is well known that one can write

$$\left\langle \frac{1}{\Sigma_t} \right\rangle = \frac{1}{\Sigma_b} \left(1 - \left\langle \frac{\Sigma_R}{\Sigma_t} \right\rangle \right) \quad (1)$$

so that $\left\langle \frac{1}{\Sigma_t} \right\rangle$ involves expressions of the same form as $\left\langle \frac{\Sigma_f}{\Sigma_t} \right\rangle$ and $\left\langle \frac{\Sigma_\gamma}{\Sigma_t} \right\rangle$.

*On temporary leave of absence from University of Wisconsin.

[1]H. H. HUMMEL and R. N. HWANG, *Technical Note, Nucl. Sci. Eng.*, this issue, p. 98.

[2]R. N. HWANG, "Doppler Effect Calculations with Interference Corrections," *Nucl. Sci. Eng.*, **21**, 523 (1965).

[3]E. A. FISCHER, Karlsruhe, Germany, private communication.

[4]J. L. ROWLANDS, AEEW-M398, unpublished UKAEA report, Winfrith (1963).

[5]P. GREEBLER, General Electric, San Jose, California, private communication.

[6]R. FROELICH and K. OTT, in press, *Nucl. Sci. Eng.*, (1965).

Froelich and Ott[6] have given a nice discussion of the significance of the effective and modified effective cross sections. They show that, if reactivity changes are to be calculated, then one should use the usual effective cross sections defined by Eqs. (1.1), and if changes in reaction rate are to be calculated, one should use the modified cross sections defined by Eqs. (1.2). It is usually of no practical interest in fast reactors to calculate the change in a reaction rate due to Doppler effect. One might concieve a difficult cross-section measurement experiment for the purpose of experimentally investigating Doppler effect, in which the change in reaction rate would be the pertinent quantity.

We believe that Hwang's Figs. 1 and 2 are misleading. They give the reduction in δk from the corresponding values calculated by the isolated resonance approximation. The reduction is that which results from accounting for overlap between all resonances of the mixture. He does not clearly explain that the reassignment of the total reactivity change among the different isotopes, which results from his use of the modified cross sections, prevents the reactivity reductions shown in his Fig. 1 from being interpreted as reductions in the Pu^{239} Doppler effect, as we have defined it above. The results are similar for U^{235} in Fig. 2. The reductions are much less when properly calculated with the normal effective cross sections.

We wish to stress that our disagreement with Hwang's interpretation of overlap effects is only when the overlapping resonances are in two different materials or sequences. He presents an original and apparently useful procedure for accounting for overlap among resonances within a given material. However, we wish to point out an important error in Eq. (2.8) which should read

$$\tilde{\sigma}_x \approx \frac{\Sigma_b}{N} \sum_{\substack{\text{spin}\\ \text{states}}} \frac{1}{\langle S \rangle_j} \sum_{m,n=1}^{\infty} (-1)^{m+n-1} \frac{(n+m-1)!}{n!(m-1)!} \times \left[\left\langle \frac{\Gamma_{xk}}{\Gamma_k} \frac{1}{\beta_k^m \beta_{k'}^n} \int_{-\infty}^{\infty} dE\, \psi_k^m \psi_{k'}^n \right\rangle \right]$$

We do not know whether this error is also reflected in the numerical results reported.

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*R. B. Nicholson**

Lawrence Radiation Laboratory
University of California
Livermore, California

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