instead of using a linear time interpolation of the shape function within an "outer" time step (as in the QS method), the interpolation, i.e., the ratio (Z_{l+1}/Z_l) at time τ_n in Eq. (7) of Ref. 1,

$$\Psi(\boldsymbol{r},\tau_n) = Z_l\Psi_l(\boldsymbol{r}) + Z_{l+1}\Psi_{l+1}(\boldsymbol{r}) \quad , \tag{2}$$

is determined variationally. The full shape function is computed only at the outer time steps as in all QS methods, the lumped parameters being calculated using the interpolated shape. The Hauss and Kastenberg method does not prescribe how to compute the full shape function at these outer time steps; apparently, in the numerical example finite difference is used. Hauss and Kastenberg claim two advantages:

1. faster convergence of outer iterations
2. use of coarser outer meshes.

It seems that point 2 is the primary motivation, although it is not stressed by Hauss and Kastenberg. When strong feedbacks are present, point 1 is not obvious in the light of the earlier remarks made in this Letter. In fact, in the presence of strong feedbacks, all the QS methods, including their synthetic variations,[1-3] will suffer from poor convergence and increased outer iterations.

In the same light, one more observation is to be made. Hauss and Kastenberg make a claim (p. 328 of Ref. 1) that the new interpolation variationally determined not only gives better values of lumped parameters (as we note) but also could provide a "next-iterate guess" of the flux shape on again reaching T_{l+1}, thus reducing the number of subsequent space-dependent calculations required at this time. Since the outer extrapolated shape $\Psi_{l+1}(\boldsymbol{r})$ at T_{l+1} is incorrect due to the lack of proper accounting of feedback effects, then by "refitting" the function by using incorrect shapes at time T_{l+1}, one cannot obtain a correct shape, especially if the correct shape is not bracketed, as will be the case in the presence of strong feedbacks. In the numerical example given by Hauss and Kastenberg, they obtain a very good convergence in the shape on reaching the outer time step T_{l+1}, obviating the need for outer iterations altogether, mainly because the nonlinear feedback effects were ignored.

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Reply to "On the Quasi-Static Synthesis Method for Space-Time Dynamics Problems"

Having reviewed the Letter by Doshi,[1] we felt it necessary to reply to several erroneous points presented there. First, we take exception with Doshi's claim that the essential features of the method presented in a paper by him and Grossman[2] are identical to those of the quasi-static synthesis (QSS) method.[3] Aside from the fact that both methods employ a combination of flux-factorization and synthesis techniques to solve the space-time equations (as do a few other methods, notably Refs. 4 and 5), there is very little that the two methods have in common. In fact, as is pointed out below, it is the very differences between the two methods that should inevitably lead to the success of the QSS method as a general approach for solving space-time kinetics problems. Furthermore, Doshi's statement that the essentials of a "quasi-static (QS) synthesis" method were originally introduced by him and Grossman is in error. The method finds its origin in the linear interpolation technique of the improved QS method developed by Meneley et al.[4] In this technique, the rapidly varying amplitude function is calculated via point kinetics, while the space-time-dependent flux shape is blended (in an *a priori* manner, i.e., linearly with respect to time) from trial shapes calculated at the inner and outer bracketing time steps. Besides this approach, Kessler[5] also employs a method that relies on a combination of the ideas of the QS and synthesis methods, although in a somewhat reversed sense. In Kessler's method, time-discontinuous synthesis is used to determine the rapidly varying time-dependent amplitude functions, while flux shapes are obtained from an iterative solution of the QS shape function equation. Thus, it should be clear that the essentials of a method that somehow combines the ideas of flux-factorization and time-synthesis have existed prior to the paper by Doshi and Grossman. In this regard, it should also be pointed out that the title of our Note,[3] "Introduction of the Quasi-Static Synthesis Method for . . . ," is not meant to imply an introduction of the general idea of combining QS and synthesis methods, but rather the introduction of the specific method (details included) proposed in the Note.

It is incorrect to compare the method developed by Doshi and Grossman to the QSS method, since the aims of the two methods are of a completely different nature. The former technique is specifically tailored to solve a restrictive (if not unrealistic) transient, that is, one in which the reactor properties over all but a small segment of the core remain unchanged for the duration of the transient. The QSS method, on the other hand, is an attempt at developing a general approximate method that is applicable to a wide range of space-time dynamics problems. However, since Doshi appears to be representing the method developed by him and Grossman as a general approach for solving the space-time equations, a brief reexamination of the method is in order. The class of expansion functions that Doshi chooses to describe the time-dependent flux shape over the *bulk*[6] of the reactor core consists of the eigenfunctions of a single operator equation,

$$L_j\,{}_j\Phi_{k,n} = {}_j\lambda_{k,n}\,{}_j\Phi_{k,n} \quad , \tag{1a}$$

where

$$L_j = \nabla^2 + \Sigma_j/D_j \quad , \quad (j = 1, 2, \ldots, G) \quad . \tag{1b}$$

The D_j's and Σ_j's in this single equation are those corresponding to the initially critical reactor state. The main problem with this technique is in being able to accurately represent flux shapes that occur during the transient using a reasonable number of expansion functions. This is especially true if

[1] J. B. DOSHI, *Nucl. Sci. Eng.*, **71**, 343 (1979).

[2] J. B. DOSHI and L. M. GROSSMAN, *Nucl. Sci. Eng.*, **65**, 106 (1978).

[3] B. I. HAUSS and W. E. KASTENBERG, *Nucl. Sci. Eng.*, **69**, 326 (1979).

[4] D. A. MENELEY et al., "Fast-Reactor Kinetics–The QX-1 Code," ANL-7769, Argonne National Laboratory (1971).

[5] G. KESSLER, *Nucl. Sci. Eng.*, **41**, 115 (1970).

[6] A "pulse" function $f_j(z,t)$ is used to reproduce the localized change in reactor properties.

reactor properties change significantly, as is the case in the presence of strong reactivity feedback. As pointed out by Henry,[7] if approximate fluxes for a range of operating conditions are to be found by the synthesis procedure, it is intuitively unappealing to generate expansion functions using an operator equation corresponding to only one condition in this range. Thus, this procedure is not recommended for most practical problems. The question then arises: How should expansion functions be chosen? The obvious answer is to pick functions that bracket the spatial/spectral shapes expected throughout the transient, that is, instead of generating expansion functions using an operator corresponding to a single reactor state, use operators bracketing a range of conditions expected during the transient. It was with this in mind that one of the authors (BIH) advised Doshi (during his stay at the University of California at Los Angeles as a post-doctoral fellow) that a set of bracketing expansion functions might be obtained by using the fundamental λ modes corresponding to various hypothetical reactor configurations anticipated during the transient. There are, however, several problems associated even with this procedure. First, since various reactor types span a wide range of dynamic response characteristics, it is extremely difficult (i.e., requiring a certain amount of clairvoyance) to determine an *a priori* set of reactor configurations that bracket those occurring during the transient. Furthermore, because all expansion functions are used in the calculations at each time step, even though many of these trial functions may be inappropriate at this specific time, the synthesis technique can become unnecessarily expensive, approaching that of a full finite difference calculation. Although this second difficulty can to some extent be obviated by employing a discontinuous time-synthesis approach,[8] the type of synthesis method proposed by Doshi does not lend itself well to transients where reactor properties (and/or flux shapes) change significantly with time. In contrast to this procedure, note that the QSS method uses only the inner and outer trial shapes of a given major time interval to synthesize shape functions within this interval. Since these bracketing trial shapes are calculated as the transient progresses (rather than being chosen in some *a priori* manner at the beginning of the transient), one is fairly confident of obtaining reasonably good representations of shape functions within the major time interval.

In his Letter, Doshi[1] asserts that in the presence of strong feedback the advantages of the QSS method over the conventional QS method become negligible. His main arguments for this conclusion are given below.

1. When reactor properties are initially "extrapolated" to the outer time T_{l+1}, one does not have knowledge of the changes in these properties due to feedbacks which are not yet evaluated. Thus, in the presence of strong feedback, the initial outer trial shape calculated from these incorrect reactor properties will be grossly in error and may no longer bracket the true flux shape.

2. Since the initial outer trial shape at T_{l+1} (i.e., Ψ_{l+1}) may be incorrect, due to the lack of proper accounting of feedback effects, then any synthesized flux shapes (and conditions that they generate via the reactivity parameters) will also be incorrect, leading to an increased number of outer iterations. Because of this increase in the number of outer iterations, Doshi concludes that the QSS method will approach that of the conventional QS method in terms of computational effort.

In answer to the arguments Doshi raises, the following comments seem appropriate.

1. As stated in our Note,[3] the diffusion theory parameters needed for the initial detailed solution at the outer time step (T_{l+1}) are found from a "fixed-shape" point kinetics calculation of the core. This is done by assuming that the flux shape at the inner time T_l (i.e., Ψ_l) persists throughout the major time interval. Since this constant flux shape assumption is the only approximation made and since the major time interval comprises only a small part of the entire transient, one should obtain a fairly accurate estimate of spatial feedbacks occurring over the time interval. Thus, Doshi's assertion that we have very limited (if any) knowledge of the feedback effects at the extrapolated time T_{l+1} is incorrect. Also, the initial outer trial shape (i.e., Ψ_{l+1}), while not exact, should be fairly close to the true flux shape at this time.

2. To answer the second point Doshi raises, consider Fig. 9 in Ref. 3, which illustrates the extrapolation capabilities of the QSS method. In this figure, the inner trial shape at 0.25 s is identical with the exact flux shape, while the (initial) outer trial shape at 0.5 s is grossly in error, being much less tilted than the true shape at this time. Note, however, that the synthesized flux shape at 0.39 s (which is more tilted than either the inner or outer flux shapes) is still very close to the corresponding exact shape. This shows that if the outer bracketing flux shape (which depends to some extent on approximate conditions at the outer time step) is somewhat in error and/or does not bracket all shape functions within the major time interval, the method has enough flexibility to give synthesized flux shapes that are still quite good. Thus, at given times within the major time interval, the QSS method combines the inner and outer trial shapes such that they best represent the true shape functions at these times. Because of this, a vast improvement on flux shapes (and conditions that they generate via the reactivity parameters) can be obtained at times within the major time interval. In contrast, the linear interpolation procedure of the conventional QS method requires that shape functions within the major time interval vary *linearly* with respect to time between the inner and outer trial shapes. As a result, the method affords no correction on the flux shape within the major time interval but only after another full trial shape calculation has been performed at the outer time step. Therefore, if reactor properties extrapolated to the outer time step are grossly in error, so that the resulting initial outer trial shape is "incorrect," one would expect that the number of iterations required for convergence of the QSS method would be *much* less than that required for convergence of the conventional QS method. Furthermore, although the number of iterations for either method will increase in the presence of strong spatially dependent feedback (where flux shapes can vary in a highly nonlinear fashion with time), this increase should be much smaller for the QSS method than for the conventional method.

Another important point not yet stressed relates directly to the linear interpolation approximation of the QS method. In those situations where the true flux shape changes in a nonlinear manner with respect to time, it seems obvious that no matter how accurately trial shapes are known at the bracketing time steps (i.e., T_l and T_{l+1}), shape functions generated (via the linear interpolation approximation) at

[7] A. F. HENRY, *Nuclear Reactor Analysis*, MIT Press, Cambridge, Massachusetts (1975).

[8] J. B. YASINSKY, *Nucl. Sci. Eng.*, **29**, 381 (1967).

times within the major time interval will necessarily be in error. This arises because the shapes at these intermediate times are not a result of the true reactor conditions but rather are determined from the *a priori* mixing (i.e., linear with respect to time) of the bracketing trial shapes. Thus, convergence of the QS method to correct solutions seems rather questionable for these situations. This problem is not anticipated with the QSS method because intermediate shape functions are not constrained to vary linearly between the inner and outer trial shapes, but only to satisfy the true reactor conditions (in a weighted integral sense) at the times in question.

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