Letters to the Editor

On the Quasi-Static Synthesis Method for Space-Time Dynamics Problems

In a recent Note, Hauss and Kastenberg[1] presented the quasi-static synthesis (QSS) method of solution for space-time reactor kinetics problems, which blends the concepts of the quasi-static (QS) method and the synthesis method. In this Letter, I would like to make some observations and remarks about the above Note. At the outset, I want to point out that the word "Introduction" used in the title of the above Note is misleading, as this method was already introduced earlier by myself and Grossman.[2] Although the method was not formally named in our paper, the essential features are identical to the QSS method of Hauss and Kastenberg.[1] There are, however, differences in some of the details that I would like to address in the following.

Essentially, both methods of calculation are based on the fact that the principle of the flux factorization technique, in the QS method, in which the slowly varying flux shape is factored out and calculated less frequently than the rapidly varying amplitude function calculated by the "point kinetics" approach and the principle of the synthesis method, in which the space-time-dependent flux shape at different time points is synthesized or blended from some trial functions, can together be profitably employed to significantly reduce the computation time for a certain class of problems. The difference in our methods lies in the choice of trial functions to generate the "true" shape function.

In our method,[2] after the formal reduction of the QS method,[3,4] we represented the space- and time-dependent flux shape by Eq. (35), i.e.,

$$\psi_j(r,\theta,z,t) = \cos\left(\frac{\pi z}{2H}\right) \sum_{n=0}^{\infty} \sum_{k=1}^{\infty} {}_jZ_{k,n}(t)\,{}_j\bar{\phi}_{k,n} + f_j(z,t) \quad , \qquad (1)$$

where ${}_j\bar{\phi}_{k,n}$ is the orthonormal eigenfunction of the group removal operator L_j satisfying all the interface and boundary conditions. The function $f_j(z,t)$ is a localized [in the (r,θ) plane] "pulse" function to reproduce the localized change in reactor properties for the problem considered. Thus, the trial functions consisted of satationary eigenfunctions, ${}_j\bar{\phi}_{k,n}$, determined *a priori* and the pulse function, $f_j(z,t)$, calculated using the perturbation formulation at each intermediate time step t_i^n (Fig. 8 of Ref. 2) and used in the calculation of lumped parameters ρ, $\bar{\beta}$, etc. The mixing coefficients, ${}_jZ_{k,n}$, are evaluated only at the outer time steps t_i^B and are linearly interpolated in between in the spirit of the QS method.[3] This approach is best suited and was used for the problem involving only localized [in the (r,θ) plane] changes in the reactor properties, since in that situation, the variation in the flux shape will be mainly confined to the vicinity of the disturbances warranting a more frequent evaluation of the pulse function $f_j(z,t)$ (at the intermediate time steps t_i^n), with a rather slow change in the bulk of the reactor core away from the location of the disturbances justifying an infrequent calculation of ${}_jZ_{k,n}$ (at outer time steps t_i^B).

Thus, the problem of solving for the space-time-dependent shape function was reduced to evaluating $f_j(z,t)$ at different times and solving for the mixing coefficient ${}_jZ_{k,n}$ that satisfied algebraic coupled equations at a given time point (approximating the time derivative by the usual implicit finite difference as in the "improved QS method"[3]). Thus, the computational effort was reduced tremendously even while obtaining a qualified three-dimensional shape function solution specific to the problem considered.

In contrast, Hauss and Kastenberg[1] have employed the calculated flux shapes at fixed outer time steps (see Fig. 1 of Ref. 1 or Fig. 8 of Ref. 2) as the trial functions that are blended to obtain the true flux shape in between these time points. This approach has several merits and demerits *vis-à-vis* our approach.[2] First, since the shape function at the outermost time step (T_{I+1} in Fig. 1 of Ref. 1) is calculated using the extrapolated reactor composition at that time, it is likely to be very accurate for problems where external changes in reactor properties (i.e., due to externally affected changes in cross section, etc.) dominate over the changes due to feedback during the course of the transient. This is obvious due to the fact that when the reactor properties are extrapolated in time, one does not have a good knowledge of the changes in the properties due to feedbacks that are not as yet evaluated. On the other hand, for the problems where the externally affected changes in the reactor properties are rather small and the transient proceeds "on its own," such an extrapolated shape function calculation can be in gross error and will no longer bracket the true shape. This just means more outer iterations are required to arrive at the convergent solution, if it is at all possible to do so.

This is where the computational effort in the approach of Hauss and Kastenberg becomes more expensive and the advantage of the "synthesis" part in the QSS is lost, since, if the number of outer iterations is increased, the method of Hauss and Kastenberg will approach the conventional QS method[3] in terms of computational effort, since we recall that the conventional QS method is nothing but one particular type of blending of two outer shapes, i.e., linear interpolation in time. Thus, the only point made in Ref. 1 is that

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[1]B. I. HAUSS and W. E. KASTENBERG, *Nucl. Sci. Eng.*, **69**, 326 (1979).

[2]J. B. DOSHI and L. M. GROSSMAN, *Nucl. Sci. Eng.*, **65**, 106 (1978).

[3]D. A. MENELEY et al., "Fast-Reactor Kinetics–The QX-1 Code," ANL-7769, Argonne National Laboratory (1971).

[4]D. A. MENELEY et al., "A Kinetic Model for Fast Reactor Analysis in Two Dimensions," in *Proc. Mtg. Dynamics of Nuclear Systems*, Tucson, Arizona, University of Arizona Press (1972).

instead of using a linear time interpolation of the shape function within an "outer" time step (as in the QS method), the interpolation, i.e., the ratio (Z_{l+1}/Z_l) at time τ_n in Eq. (7) of Ref. 1,

$$\Psi(\boldsymbol{r},\tau_n) = Z_l\Psi_l(\boldsymbol{r}) + Z_{l+1}\Psi_{l+1}(\boldsymbol{r}) \quad , \tag{2}$$

is determined variationally. The full shape function is computed only at the outer time steps as in all QS methods, the lumped parameters being calculated using the interpolated shape. The Hauss and Kastenberg method does not prescribe how to compute the full shape function at these outer time steps; apparently, in the numerical example finite difference is used. Hauss and Kastenberg claim two advantages:

1. faster convergence of outer iterations
2. use of coarser outer meshes.

It seems that point 2 is the primary motivation, although it is not stressed by Hauss and Kastenberg. When strong feedbacks are present, point 1 is not obvious in the light of the earlier remarks made in this Letter. In fact, in the presence of strong feedbacks, all the QS methods, including their synthetic variations,[1-3] will suffer from poor convergence and increased outer iterations.

In the same light, one more observation is to be made. Hauss and Kastenberg make a claim (p. 328 of Ref. 1) that the new interpolation variationally determined not only gives better values of lumped parameters (as we note) but also could provide a "next-iterate guess" of the flux shape on again reaching T_{l+1}, thus reducing the number of subsequent space-dependent calculations required at this time. Since the outer extrapolated shape $\Psi_{l+1}(\boldsymbol{r})$ at T_{l+1} is incorrect due to the lack of proper accounting of feedback effects, then by "refitting" the function by using incorrect shapes at time T_{l+1}, one cannot obtain a correct shape, especially if the correct shape is not bracketed, as will be the case in the presence of strong feedbacks. In the numerical example given by Hauss and Kastenberg, they obtain a very good convergence in the shape on reaching the outer time step T_{l+1}, obviating the need for outer iterations altogether, mainly because the nonlinear feedback effects were ignored.

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Reply to "On the Quasi-Static Synthesis Method for Space-Time Dynamics Problems"

Having reviewed the Letter by Doshi,[1] we felt it necessary to reply to several erroneous points presented there. First, we take exception with Doshi's claim that the essential features of the method presented in a paper by him and Grossman[2] are identical to those of the quasi-static synthesis (QSS) method.[3] Aside from the fact that both methods employ a combination of flux-factorization and synthesis techniques to solve the space-time equations (as do a few other methods, notably Refs. 4 and 5), there is very little that the two methods have in common. In fact, as is pointed out below, it is the very differences between the two methods that should inevitably lead to the success of the QSS method as a general approach for solving space-time kinetics problems. Furthermore, Doshi's statement that the essentials of a "quasi-static (QS) synthesis" method were originally introduced by him and Grossman is in error. The method finds its origin in the linear interpolation technique of the improved QS method developed by Meneley et al.[4] In this technique, the rapidly varying amplitude function is calculated via point kinetics, while the space-time-dependent flux shape is blended (in an *a priori* manner, i.e., linearly with respect to time) from trial shapes calculated at the inner and outer bracketing time steps. Besides this approach, Kessler[5] also employs a method that relies on a combination of the ideas of the QS and synthesis methods, although in a somewhat reversed sense. In Kessler's method, time-discontinuous synthesis is used to determine the rapidly varying time-dependent amplitude functions, while flux shapes are obtained from an iterative solution of the QS shape function equation. Thus, it should be clear that the essentials of a method that somehow combines the ideas of flux-factorization and time-synthesis have existed prior to the paper by Doshi and Grossman. In this regard, it should also be pointed out that the title of our Note,[3] "Introduction of the Quasi-Static Synthesis Method for . . . ," is not meant to imply an introduction of the general idea of combining QS and synthesis methods, but rather the introduction of the specific method (details included) proposed in the Note.

It is incorrect to compare the method developed by Doshi and Grossman to the QSS method, since the aims of the two methods are of a completely different nature. The former technique is specifically tailored to solve a restrictive (if not unrealistic) transient, that is, one in which the reactor properties over all but a small segment of the core remain unchanged for the duration of the transient. The QSS method, on the other hand, is an attempt at developing a general approximate method that is applicable to a wide range of space-time dynamics problems. However, since Doshi appears to be representing the method developed by him and Grossman as a general approach for solving the space-time equations, a brief reexamination of the method is in order. The class of expansion functions that Doshi chooses to describe the time-dependent flux shape over the *bulk*[6] of the reactor core consists of the eigenfunctions of a single operator equation,

$$L_j\,{}_j\Phi_{k,n} = {}_j\lambda_{k,n}\,{}_j\Phi_{k,n} \quad , \tag{1a}$$

where

$$L_j = \nabla^2 + \Sigma_j/D_j \quad , \quad (j = 1, 2, \ldots, G) \quad . \tag{1b}$$

The D_j's and Σ_j's in this single equation are those corresponding to the initially critical reactor state. The main problem with this technique is in being able to accurately represent flux shapes that occur during the transient using a reasonable number of expansion functions. This is especially true if

[1] J. B. DOSHI, *Nucl. Sci. Eng.*, **71**, 343 (1979).

[2] J. B. DOSHI and L. M. GROSSMAN, *Nucl. Sci. Eng.*, **65**, 106 (1978).

[3] B. I. HAUSS and W. E. KASTENBERG, *Nucl. Sci. Eng.*, **69**, 326 (1979).

[4] D. A. MENELEY et al., "Fast-Reactor Kinetics–The QX-1 Code," ANL-7769, Argonne National Laboratory (1971).

[5] G. KESSLER, *Nucl. Sci. Eng.*, **41**, 115 (1970).

[6] A "pulse" function $f_j(z,t)$ is used to reproduce the localized change in reactor properties.