

a cladded radioactive surface are essentially the same, Kappos' conclusions should extend to Romero's work.

Romero remarks that the extension of the method to electrons may be complicated by electron scattering. It has been shown that the straight travel path approximation may be retained for electron energies < 200 keV. In particular, Charlton and Cormack have shown that highly accurate results are obtained when the electrons are assumed to have a distribution of ranges which is a function of the total electron path length.⁵

A general survey of the methods used in analyzing the bone/soft tissue dosimetry problem may be found in Spiers.⁶ These methods should be useful to anyone studying energy transport from cladded radioactive surfaces.

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⁵D. E. CHARLTON and D. V. CORMACK, in *Proc. Symp. Microdosimetry Ispra (Italy)*, November 13-15, 1967, p. 413, H. G. EBERT, Ed., European Communities, Brussels (1968).

⁶F. W. SPIERS, *Radioisotopes in the Human Body: Physical and Biological Aspects*, Chap. 5, Academic Press, New York and London (1968).

Reply to Comments on Residual Power Fluxes from Radioactive Surfaces

In addition to pointing out the similarity of my formulation with that of Charlton and Cormack, Schlenker has made some worthwhile extensions to the work. Although the formulation cited approaches the problem from a slightly different point of view, that of energy deposition at a point (dose), as shown by Schlenker, it can be used to calculate fractional residual power fluxes and can be readily extended to cylindrical and spherical geometries. The work of Charlton and Cormack apparently does not consider the case of thin layers; i.e., $L_c + L_l < 1$; however, this extension is given by Schlenker in his comments. A minor disadvantage of the method suggested is that it requires numerical integration of the geometrical factors to obtain fractional power fluxes, while in Romero's formulation these can be read directly (e.g., Fig. 4). Romero's method also gives a direct calculation of particle fluxes and energies.

To some extent the work of Kappos supports the contention that the approximate range-energy relationship gives accurate results in power calculations. However, these results should perhaps not be extended outright to clads differing substantially from tissue. Some clads considered for high temperature applications include heavy refractory materials which may show some variation of the range with atomic mass.¹ While it is not known how this would affect the accuracy of power flux calculations, perhaps a final judgment should await further analysis.

¹S. E. LIVERHANT, *Elementary Introduction to Nuclear Physics*, p. 330, John Wiley and Sons, New York (1960).

The comment that the straight-ahead approximation has been extended to electrons by Charlton and Cormack could also represent a substantial extension of the applicability of this method. I was not able to obtain a copy of the reference cited in time to review here, so I will comment on some results obtained with beta sources. Calculations made for various beta sources showed considerable deviation from the straight-ahead approximation. In particular, predicted residual particle fluxes were overestimated by factors of 2 to 3 for soft betas from the theory presented by Libby.² It was concluded, perhaps prematurely, that electron scattering precluded the application of this method. Contradiction with the reported work of Charlton and Cormack may indicate that accurate results for power fluxes, which were not compared with existing data, still may be obtained by this method even though particle fluxes may be in error. In any case, further investigation of this discrepancy is warranted, especially in view of substantial extension of the method should its applicability be verified.

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²W. F. LIBBY, *Anal. Chem.*, **19**, 2 (1947).

Comments on Certain Instabilities in the "Initial Value" Problem

It was shown¹ that predicting the time evolution of an initial pulse in a moderator, i.e., solving the "initial value" problem, is a question that can be transferred to studying certain features of the Laplace transform technique. This study displays some difficulties such as

1. proving the existence and uniqueness of the solution in the energy-dependent case
2. the knowledge of the true scattering kernel and, implicitly, of the analytical structure of the transformed distribution $\tilde{n}(\mathbf{r}, \mathbf{v}, \lambda)$ in the complex λ plane.

In connection with this second aspect we wish to point out some remarks. On principle, knowing $\Sigma_s(\mathbf{v}', \mathbf{v})$, we are able to obtain the distribution $n(\mathbf{r}, \mathbf{v}, t)$ by solving the transport equation by the method indicated in Ref. 1. This solution assumes that it is possible to deform the Bromwich contour in the complex λ plane and to account for all the singularities of $\tilde{n}(\mathbf{r}, \mathbf{v}, \lambda)$. This is possible only when $\Sigma_s(\mathbf{v}', \mathbf{v})$ is known; otherwise, we do not have an analytical formula for $\tilde{n}(\mathbf{r}, \mathbf{v}, \lambda)$ and there is no possibility of continuing it in the left halfplane.

Let \tilde{n}_λ be the solution of the transport equation with the kernel Σ_s and \tilde{n}'_λ the solution with the kernel Σ'_s . Because of the analyticity, if Σ_s and Σ'_s differ slightly, \tilde{n}_λ and \tilde{n}'_λ will also differ slightly in certain right half planes, but they could differ greatly in the left half plane. We are not able to tell *a priori* whether \tilde{n}_λ and \tilde{n}'_λ , and hence $n(t)$ and $n'(t)$, will differ slightly or greatly. Lack of knowledge of the analytic formula for Σ_s makes this problem unpredictable.

¹I. KUŠČER, *IAEA Symp. on Neutron Thermalization and Reactor Spectra*, Ann Arbor, Michigan (1967).

On the contrary, we can prove that a soluble model such as Corngold's² is stable against small changes of $\Sigma_s(v)$ eventually due to experimental uncertainties in measuring the cross sections. The model used by Corngold,

$$v' \Sigma_s(v', v) = \beta v' \Sigma_s(v') v \Sigma_s(v) M(v) ,$$

leads to

$$\tilde{\eta}_\lambda = \beta M(v) \frac{v \Sigma_s(v) v_0 \Sigma_s(v_0)}{\lambda + v_0 \Sigma(v_0) + iBv_0 \mu_0} \frac{g(v, \lambda)}{1 - \beta b(\lambda)} + \delta(v - v_0) \frac{1}{\lambda + v \Sigma(v) + iBv \mu_0}$$

with

$$g(v, \lambda) = \frac{1}{2iBv} \ln \frac{\lambda + v \Sigma + iBv}{\lambda + v \Sigma - iBv}$$

$$\tilde{b}(\lambda) = \int_0^\infty (v \Sigma_s)^2 M(v) g(v, \lambda) dv .$$

Let $\Sigma'_s(v)$ differ from $\Sigma_s(v)$ by an error $\epsilon(v)$ which is constant for simplicity: $\Sigma'_s(v) = \Sigma_s(v) + \epsilon$. Then

$$b'(\lambda) = b(\lambda) + \int_0^\infty dv (2v^2 M \Sigma_s \epsilon + v^2 \epsilon^2 M) g(v, \lambda) = b(\lambda) + \eta_\epsilon(\lambda) \lim_{\epsilon \rightarrow 0} \eta_\epsilon(\lambda) \rightarrow 0$$

and $\delta \tilde{\eta}_\lambda = \tilde{\eta}'_\lambda - \tilde{\eta}_\lambda$ tends to zero when ϵ tends to zero, uniformly in λ .

It would probably be interesting to find the largest class of functions for the sections for which the inverse Laplace transform would be a stable procedure against the small (experimental) errors; however, because we ignore the true scattering kernel in regard to its analytic formula and we have it given at most by a histogram, we do not believe that one can find a function among the elements of this class that could be "brought closer to reality."³ At most, this class will be a class of mathematical rather than physical models.

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²N. CORNGOLD, *Nucl. Sci. Eng.*, **19**, 80 (1964).

³R. CONN and N. CORNGOLD, *Nucl. Sci. Eng.*, **37**, 85 (1969).

Corrigendum

T. C. CHAWLA and B. M. HOGLUND, "Pressure Distribution for Axisymmetric Two-Dimensional Flow in a Plenum During Coolant Explosion," *Nucl. Sci. Eng.*, **43**, 87 (1971).

On p. 89, Eqs. (17a) and (17b) are to be replaced by the following single equation that is applicable for both cases, viz., $m \neq n$ and $m = n$:

$$P_{av}(0, t) = \rho g H + \rho \left[H + \frac{4}{b^2} \sum_{n=1}^{\infty} \frac{J_1^2(\alpha_n b)}{\alpha_n^2 J_0^2(\alpha_n a)} \right] \frac{d}{dt} \left(U_0 \frac{b^2}{a^2} \right) - \frac{1}{2} \rho \left\{ \frac{8b^2}{a^4} U_0^2 \sum_{n=1}^{\infty} \frac{J_1^2(\alpha_n b)}{\alpha_n^2 J_0^2(\alpha_n a)} \left[J_0^2(\alpha_n b) + J_1^2(\alpha_n b) - \frac{1}{\alpha_n b} J_0(\alpha_n b) J_1(\alpha_n b) \right] \right\} - \frac{1}{2} \rho \left\{ \frac{16b}{a^4} U_0^2 \sum_{m=1}^{\infty} \sum_{n=m+1}^{\infty} \frac{J_1(\alpha_m b) J_1(\alpha_n b)}{\alpha_m \alpha_n (\alpha_m - \alpha_n) J_0^2(\alpha_m a) J_0^2(\alpha_n a)} \right\} \times [J_1(\alpha_m b) J_0(\alpha_n b) - J_0(\alpha_m b) J_1(\alpha_n b)] + \frac{8b^2}{a^4} U_0^2 \sum_{n=1}^{\infty} \frac{J_1^2(\alpha_n b)}{\alpha_n^2 J_0^2(\alpha_n a)} \left. \right\} + P_a . \quad (17)$$