Letters to the Editor

Comments on Residual Power Fluxes from Radioactive Surfaces

I should like to point out similarities between the calculations of Romero[1] and certain radiation dose-rate calculations.

Charlton and Cormack calculated the rate of energy deposition/unit volume in a layer of soft tissue adjacent to a plane surface of bone.[2] They assumed that an α-emitter was uniformly distributed throughout the bone volume and adopted a range-energy relationship identical to the one used by Romero. In their development the bone serves as the radioactive deposit and the soft tissue serves as the clad.

Charlton and Cormack find that the rate of energy deposition/unit volume at a point a perpendicular distance x from the interface is

$$\frac{S_v E}{(R_C/R_I)}\, P_\alpha\left(\frac{x}{R_C}\right) , \qquad L_I + L_C \geq 1 ,$$

where $P_\alpha(x/R_C)$ is a function that takes into account the geometry of the calculation, and S_v, E, R_C, R_I, L_C, and L_I are the parameters used by Romero. In terms of $P_\alpha(x/R_C)$, the fractional residual power flux f_P, calculated by Romero, is

$$f_P = \frac{1}{L_I}\int_{L_C}^{1} P_\alpha(y)\,dy , \quad L_C + L_I \geq 1$$

and

$$f_P = \frac{1}{L_I}\left\{\int_{L_C}^{1} P_\alpha(y)\,dy - \int_{L_C+L_I}^{1} P_\alpha(y)\,dy\right\} , \qquad L_C + L_I < 1 ,$$

where $y = x/R_C$. Howarth has given an extensive tabulation of $P_\alpha(x/R_C)$ and it is quite easy to compute f_P by numerically integrating the tabulated values[3] of $P_\alpha(x/R_C)$. As an example, Romero quotes $f_P = 0.075$ for $L_I = 1$, $L_C = 0.2$; using the trapezoidal rule with Howarth's tables gives $f_P = 0.075524$, in agreement with Romero. Since Romero has given no tabulation for f_P, a tabulation may easily be constructed from Howarth's tables.

In addition to $P_\alpha(x/R_C)$, Howarth gives extensive tabulations of $C_\alpha(x/R_C, D_I/R_C)$ and $S_\alpha(x/R_C, D_I/R_C)$, the geometrical functions that describe emissions from the interior surfaces of cylindrical and spherical radioactive shells. In these functions, x is distance measured along a perpendicular to the inner surface of the radioactive layer and D_I is the diameter of the inner surface of that layer. As with the plane interface, f_P may be formulated quite simply and computed by interpolation and numerical integration of Howarth's values. Let the reduced thicknesses of the clad and of the radioactive deposit measured normal to the surface be L_C and L_I, let V_I denote the volume of the radioactive deposit, let z denote D_I/R_C, H the height of the cylinder, and U and W the upper limits of integration. The equations for f_P are then

Cylinder:

$$f_P = 2\pi\,\frac{R_I R_C}{(V_I/H)}\int_{L_C}^{U} C_\alpha(y,z)\left(\frac{z}{2} - y\right)dy , \qquad L_I + L_C \geq 1 ,$$

$$f_P = 2\pi\,\frac{R_I R_C}{(V_I/H)}\left\{\int_{L_C}^{U} C_\alpha(y,z)\left(\frac{z}{2} - y\right)dy - \int_{L_C+L_I}^{W} C_\alpha(y, z+2L_I)\left(\frac{z}{2} + L_I - y\right)dy\right\} , \qquad L_I + L_C < 1 ; \qquad (1)$$

Sphere:

$$f_P = 4\pi\,\frac{R_I R_C^2}{V_I}\int_{L_C}^{U} S_\alpha(y,z)\left(\frac{z}{2} - y\right)^2 dy , \qquad L_I + L_C \geq 1 ,$$

$$f_P = 4\pi\,\frac{R_I R_C^2}{V_I}\left\{\int_{L_C}^{U} S_\alpha(y,z)\left(\frac{z}{2} - y\right)^2 dy - \int_{L_C+L_I}^{W} S_\alpha(y, z+2L_I)\left(\frac{z}{2} + L_I - y\right)^2 dy\right\} , \qquad L_I + L_C < 1 . \qquad (2)$$

The upper limits of integration are

Range of z, L_I	U	W
$\frac{z}{2} \geq 1$	1	1
$\frac{z}{2} < 1$, $\frac{z}{2} + L_I \geq 1$	$\frac{z}{2}$	1
$\frac{z}{2} < 1$, $\frac{z}{2} + L_I < 1$	$\frac{z}{2}$	$\frac{z}{2} + L_I$.

At the close of his article Romero suggests that a comparison of results calculated from the approximate range-energy relationship with results calculated from a more exact relationship would be valuable in estimating the accuracy of the method. Such a comparison has been made by Kappos for the bone/soft tissue problem.[4] Kappos found that a more exact formulation of the energy-loss relationship yielded only a slight alteration in the results. Since the problem of soft tissue covering bone and the problem of

[1] JACOB B. ROMERO, *Nucl. Sci. Eng.*, **42**, 49 (1970).

[2] D. E. CHARLTON and D. V. CORMACK, *Brit. J. Radiol.*, **35**, 473 (1962).

[3] J. L. HOWARTH, *Brit. J. Radiol.*, **38**, 51 (1965).

[4] A. D. KAPPOS, *Biophysik*, **4**, 137 (1967).

a cladded radioactive surface are essentially the same, Kappos' conclusions should extend to Romero's work.

Romero remarks that the extension of the method to electrons may be complicated by electron scattering. It has been shown that the straight travel path approximation may be retained for electron energies <200 keV. In particular, Charlton and Cormack have shown that highly accurate results are obtained when the electrons are assumed to have a distribution of ranges which is a function of the total electron path length.[5]

A general survey of the methods used in analyzing the bone/soft tissue dosimetry problem may be found in Spiers.[6] These methods should be useful to anyone studying energy transport from cladded radioactive surfaces.

Robert A. Schlenker

Argonne National Laboratory
Argonne, Illinois 60439

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[5]D. E. CHARLTON and D. V. CORMACK, in *Proc. Symp. Microdosimetry Ispra (Italy)*, November 13-15, 1967, p. 413, H. G. EBERT, Ed., European Communities, Brussels (1968).

[6]F. W. SPIERS, *Radioisotopes in the Human Body: Physical and Biological Aspects*, Chap. 5, Academic Press, New York and London (1968).

Reply to Comments on Residual Power Fluxes from Radioactive Surfaces

In addition to pointing out the similarity of my formulation with that of Charlton and Cormack, Schlenker has made some worthwhile extensions to the work. Although the formulation cited approaches the problem from a slightly different point of view, that of energy deposition at a point (dose), as shown by Schlenker, it can be used to calculate fractional residual power fluxes and can be readily extended to cylindrical and spherical geometries. The work of Charlton and Cormack apparently does not consider the case of thin layers; i.e., $L_c + L_l < 1$; however, this extension is given by Schlenker in his comments. A minor disadvantage of the method suggested is that it requires numerical integration of the geometrical factors to obtain fractional power fluxes, while in Romero's formulation these can be read directly (e.g., Fig. 4). Romero's method also gives a direct calculation of particle fluxes and energies.

To some extent the work of Kappos supports the contention that the approximate range-energy relationship gives accurate results in power calculations. However, these results should perhaps not be extended outright to clads differing substantially from tissue. Some clads considered for high temperature applications include heavy refractory materials which may show some variation of the range with atomic mass.[1] While it is not known how this would affect the accuracy of power flux calculations, perhaps a final judgment should await further analysis.

The comment that the straight-ahead approximation has been extended to electrons by Charlton and Cormack could also represent a substantial extension of the applicability of this method. I was not able to obtain a copy of the reference cited in time to review here, so I will comment on some results obtained with beta sources. Calculations made for various beta sources showed considerable deviation from the straight-ahead approximation. In particular, predicted residual particle fluxes were overestimated by factors of 2 to 3 for soft betas from the theory presented by Libby.[2] It was concluded, perhaps prematurely, that electron scattering precluded the application of this method. Contradiction with the reported work of Charlton and Cormack may indicate that accurate results for power fluxes, which were not compared with existing data, still may be obtained by this method even though particle fluxes may be in error. In any case, further investigation of this discrepancy is warranted, especially in view of substantial extension of the method should its applicability be verified.

Jacob B. Romero

The Boeing Company
Seattle, Washington 98124

December 21, 1970

[2]W. F. LIBBY, *Anal. Chem.*, **19**, 2 (1947).

Comments on Certain Instabilities in the "Initial Value" Problem

It was shown[1] that predicting the time evolution of an initial pulse in a moderator, i.e., solving the "initial value" problem, is a question that can be transferred to studying certain features of the Laplace transform technique. This study displays some difficulties such as

1. proving the existence and uniqueness of the solution in the energy-dependent case
2. the knowledge of the true scattering kernel and, implicitly, of the analytical structure of the transformed distribution $\tilde{n}(\boldsymbol{r}, \boldsymbol{v}, \lambda)$ in the complex λ plane.

In connection with this second aspect we wish to point out some remarks. On principle, knowing $\Sigma_s(\boldsymbol{v}', \boldsymbol{v})$, we are able to obtain the distribution $n(\boldsymbol{r}, \boldsymbol{v}, t)$ by solving the transport equation by the method indicated in Ref. 1. This solution assumes that it is possible to deform the Bromwich contour in the complex λ plane and to account for all the singularities of $\tilde{n}(\boldsymbol{r}, \boldsymbol{v}, \lambda)$. This is possible only when $\Sigma_s(\boldsymbol{v}', \boldsymbol{v})$ is known; otherwise, we do not have an analytical formula for $\tilde{n}(\boldsymbol{r}, \boldsymbol{v}, \lambda)$ and there is no possibility of continuing it in the left halfplane.

Let $\tilde{n}_\lambda$ be the solution of the transport equation with the kernel Σ_s and $\tilde{n}'_\lambda$ the solution with the kernel Σ'_s. Because of the analyticity, if Σ_s and Σ'_s differ slightly, $\tilde{n}_\lambda$ and $\tilde{n}'_\lambda$ will also differ slightly in certain right half planes, but they could differ greatly in the left half plane. We are not able to tell *a priori* whether $\tilde{n}_\lambda$ and $\tilde{n}'_\lambda$, and hence $n(t)$ and $n'(t)$, will differ slightly or greatly. Lack of knowledge of the analytic formula for Σ_s makes this problem unpredictable.

[1]S. E. LIVERHANT, *Elementary Introduction to Nuclear Physics*, p. 330, John Wiley and Sons, New York (1960).

[1]I. KUŠČER, *IAEA Symp. on Neutron Thermalization and Reactor Spectra*, Ann Arbor, Michigan (1967).