TABLE III

Variation of $\rho$ with Parasitic Absorber Content of Fuel Plate ($0.625 \leq E \leq 1.5$ eV)

| $N$ Parasitic Absorber | $\frac{1}{1+\alpha}$ (Single-Level Formula) | $\frac{1}{1+\alpha}$ (Multilevel Formula) | $\rho$ |
|---|---|---|---|
| 0 | 0.885 | 0.815 | 0.921 ± 0.001 |
| $5 \times 10^{-5}$ | 0.407 | 0.401 | 0.986 ± 0.013 |
| $4 \times 10^{-4}$ | 0.132 | 0.129 | 0.982 ± 0.019 |
| $8 \times 10^{-4}$ | 0.0849 | 0.0856 | 1.009 ± 0.027 |

inserts constant average cross sections into each of these in such a manner as to reproduce the experimental values of the fission and absorption resonance integrals.

Actual reactor fuel elements, however, often contain parasitic absorbing isotopes, some of whose resonances may lie close to one or more of the $U^{235}$ resonances. To study the behavior of the fission-to-absorption ratio under these conditions let us consider a fuel plate of composition: $N(U^{235}) = 0.002$, $N(Zr) = 0.04$, and varying amounts of a parasitic absorber whose single-level resonance parameters $E_0 = 1.1$ eV, $\Gamma_\gamma = 67$ mV, and $\Gamma_n = 2.1$ mV are chosen to emphasize interference with the $U^{235}$ resonance at 1.14 eV. We consider once more a slab lattice composed of fuel plates and water channels, each region of which is 0.127-cm thick. Smooth fission and absorption cross sections have been added in the manner described in the last paragraph.

Values of $\rho$, the ratio of multilevel to single-level values of $1/(1+\alpha)$, are presented in Table III together with the probable errors in $\rho$ deduced from the Monte Carlo output. One notes that when parasitic absorption becomes comparable to the $U^{235}$ absorption, $\rho$ is 6% less than its initial value; thus for fuel plates of a composition typical of pressurized-water reactors the customary addition of smooth cross sections has not reproduced the correct variation of $1/(1+\alpha)$ with parasitic absorber concentration. The evaluation of the consequent errors introduced in reactor lifetime studies will have to await the availability of multilevel parameters for a wider energy range than that considered here.

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## A Semi-Empirical Description of the Detailed Thermal Flux Distribution in Uranium Oxide Clusters

At the present time there is no simple way of calculating the detailed fine structure in cluster fuel elements, although such elements are of increasing practical importance. Experimental knowledge of the fine structure in clusters is useful, therefore, for checking new calculational methods as well as for providing important design information. In design studies, as far as the thermal utilization is concerned, it is usually sufficient to know the relative *average* flux in each rod of a cluster. For thermodynamic studies, on the other hand, one often needs the two-dimensional flux distribution at each rod position, or the 'hyperfine structure.' In clusters of more than a few rods the determination of the complete hyperfine structure becomes quite tedious so that it would be useful to find an analytic expression for such distributions depending on a few easily measured quantities.

At Saclay we have measured the fine structure in a number of natural uranium oxide clusters of the type commonly used in gas-cooled, $D_2O$- or graphite-moderated reactors[1]. In two of these clusters the hyperfine structure was measured in detail.

The first was composed of 19 $UO_2$ rods of 12-mm-diam clad in 1-mm-thick aluminum. A central rod is surrounded by two concentric rings (6 rods at a radius of 19 mm and 12 at 37 mm). The second was made up of seven 22-mm-diam rods similarly clad. Six rods surround a central rod at a radius of 32 mm. Both clusters were contained in a magnesium pressure tube 106 mm in diameter.

[1] P. F. PALMEDO, "Etudes Experimentales de Structure Fine a l'Interieure des Grappes d'Oxyde $UO_2$," CEA Report No. 2387 (1964).

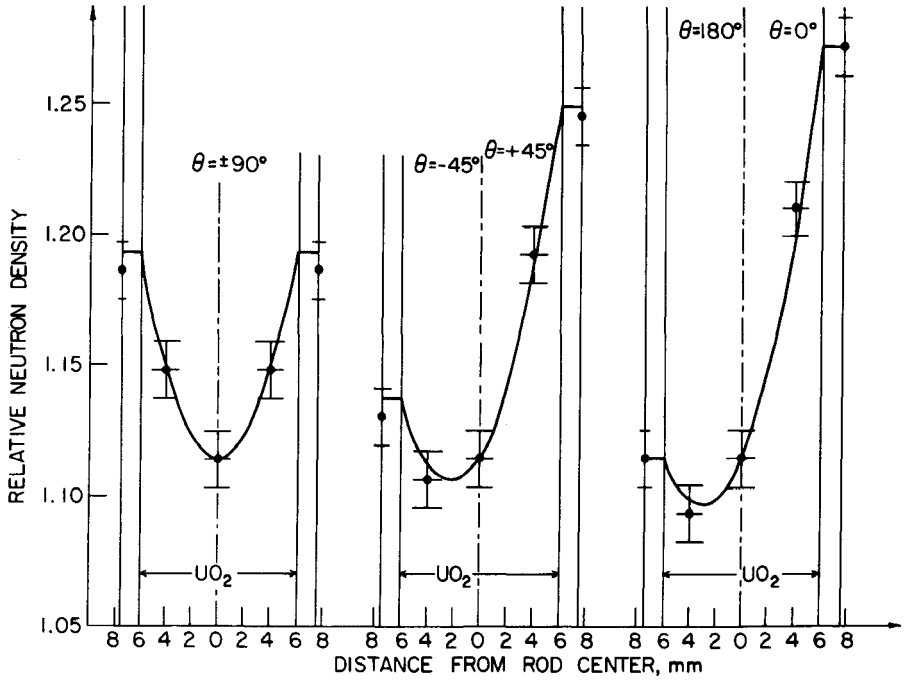


Fig. 1. Experimental and analytic neutron density in outer rod of a nineteen 12-mm rod cluster.

In both clusters it was found that an equation of the form

$$\phi_i(r, \theta) = (p_i + q_i r \cos \theta) e^{\xi_i r^2} \qquad (1)$$

described very well the flux, $\phi$, in the rods at each position, $i$, of the clusters. In the equation $r$ and $\theta$ are the polar coordinates with respect to the center of the i'th rod with $\theta = 0$ taken toward the outside of the cluster. The constants $p_i$ (indicating the flux level in rod $i$ with respect to the other rods), $q_i$ (related to the overall gradient in the cluster) and $\xi_i$ (indicating the rod flux depression) are empirically determined.

In the central rod of a cluster $q = 0$ and the flux is given by

$$\phi_0(r) = p_0 e^{\xi_0 r^2}. \qquad (2)$$

Such a flux shape was found to agree with a wide range of measurements made on single isolated rods as well as on central rods of clusters.

The value of $\xi$ usually varies only slightly throughout a cluster and may be taken to be constant. For the clusters studied in detail Eq. (1) was found to agree with the measurements at all points to within 2%.

Figure 1 shows the neutron density at various $\theta$ in an outer rod of the 19-rod cluster as measured with Dy detectors (2-mm diam) and as given by the analytical expression. The parameters used are: $p = 1.114$, $q = 0.123$ cm$^{-1}$ and $\xi = 0.190$ cm$^{-2}$.

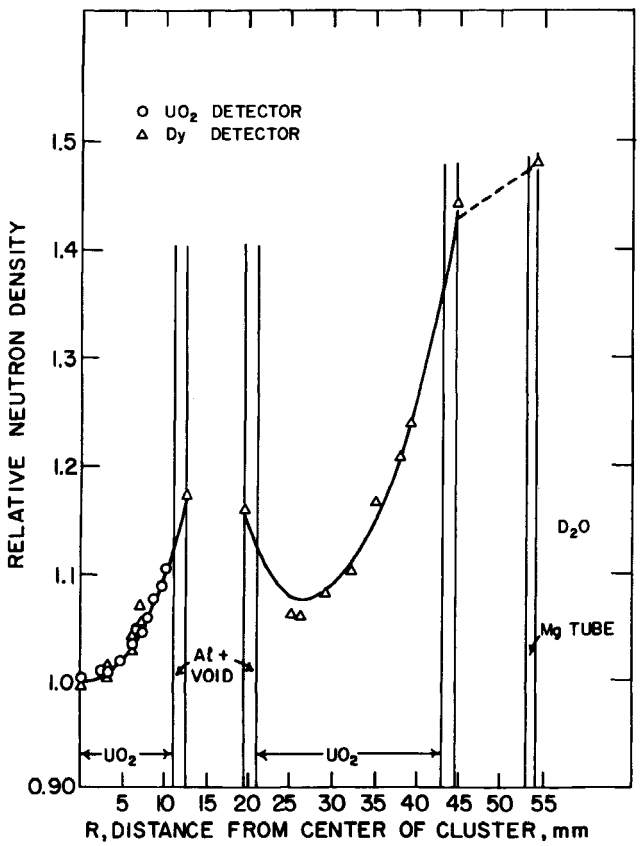


Fig. 2. Experimental and analytic neutron density in a seven 22-mm rod cluster.

Boeuf and Tassan have measured the hyperfine structure in a similar cluster when the pressure tube was filled with organic coolant. In their case, 19 natural-$UO_2$ rods of 16.2-mm diam are arranged in a hexagonal pattern. The suggested equation is again found to fit the measured distribution within the experimental errors of[2] ± 3%.

In Fig. 2 the experimental flux distribution through the 7-rod cluster is compared with the analytic function. In this case $p_0 = 1$, $p_1 = 1.103$ and a common value of $\xi = 0.098$ cm$^{-2}$ is used. As is seen, the expression can be used with an assumption of a flat flux across the cladding gap (as in Fig. 1) or a continuous flux (as in Fig. 2) or any intermediary condition depending on theoretical or experimental justification.

The suggested expression should be useful not only in reducing the amount of experimentation needed to completely specify the hyperfine structure of a given cluster, but also in extrapolating results from a given cluster to related cases.

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[2]A. BOEUF and S. TASSAN, "A Measurement of the Fine Structure Distribution of the Thermal Flux in Organic Cooled Fuel Elements in a $D_2O$ Moderated Lattice," Internal Ispra Report DPR/400, PN/37 to be published in *Energia Nucleare*.

## A Phase Integral Study of Neutron Thermalization in Graphite*

In recent years there has been a considerable amount of research[1-4] on the representation of the scattering operator by a differential operator of second order. Under the above approximation the WKB solution[5] of the diffusion equation was first introduced by Corngold[1]. He applied it to the heavy-gas model and obtained very good agreement with the 'exact' calculations. The present work is devoted to WKB calculations of the thermal-neutron spectra and other related quantities in graphite.

If the scattering operator $L$, defined by

$$L\Phi \equiv \int_0^\infty d\epsilon' \left[\Sigma_s(\epsilon' \to \epsilon)\Phi(\epsilon') - \Sigma_s(\epsilon \to \epsilon')\Phi(\epsilon)\right],$$

$$\epsilon = \frac{E}{kT},$$

is represented by a differential operator of second order, and if the neutron conservation and the detailed balance conditions are satisifed[1,2], we get

$$L = \frac{d}{d\epsilon} P(\epsilon) \left[\epsilon \frac{d}{d\epsilon} + \epsilon - 1\right].$$

If we substitute this for the scattering operator in the source-free diffusion equation and inquire about solutions of the form

$$\Phi(\epsilon, \underline{r}, t) = \exp[i\underline{B}\cdot\underline{r} + \lambda t]\Phi(\epsilon)$$

we get

$$\frac{d^2\Psi}{d\epsilon^2} + k^2(\epsilon)\Psi = 0, \tag{1}$$

where $\Psi(\epsilon) = M^{-1/2} p^{1/2} \Phi(\epsilon)$

$$-k^2(\epsilon) = \frac{1}{4}\left(\frac{q}{p}\right)^2 + \frac{1}{2}\frac{d}{d\epsilon}\left(\frac{q}{p}\right) + \left[\frac{\lambda + \lambda_a f(\epsilon)}{v_0\sqrt{\epsilon}} + D(\epsilon)B^2\right]\frac{1}{p(\epsilon)},$$

in which $\frac{\lambda_a f(\epsilon)}{v_0\sqrt{\epsilon}} = \Sigma_a(\epsilon)$

$p(\epsilon) = \epsilon P(\epsilon)$

$M(\epsilon)$ is the Maxwellian distribution $\epsilon e^{-\epsilon}$

$$q(\epsilon) = \frac{1}{M(\epsilon)}\frac{d}{d\epsilon}\left[p(\epsilon)M(\epsilon)\right]$$

$D$ is the diffusion coefficient.

and $\Phi(\epsilon)$ is the neutron flux.

If we solve the above eigenvalue equation for given values of $B^2$ and $\Sigma_a(\epsilon)$ and seek the eigenvalues and the corresponding eigenfunctions, we get quantities of interest for the interpretation of pulsed neutron experiment. Further, if we set $\lambda$=0 and solve Eq. (1) with an appropriate source term, we get the steady state spectra with leakage

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[3]H. H. W. PITCHER, "The Generalized Heavy Free Gas Thermalization Operator," AEEW-M350 (1963).
[4]M. CADILHAC *et al.*, *Proc. BNL Conf.*, II, 439 (1962).

[5]See, for example, P. M. MORSE and H. FESHBACH, "Methods of Theoretical Physics," McGraw-Hill, New York (1953); or J. HEADING, "Phase Integral Methods," John Wiley, New York (1962).