$$N(t) = N_0(1 + N'(t))$$

where N_0 is the expectation of $N(t)$ and $N'(t)$ is a random function of expectation zero. Then, using Eqs. (A.2) and (A.4), we have

$$\rho_N(\tau') = N_0^2 + \rho_{N'}(\tau') \quad ,$$

and

$$\rho_{N,\psi}(\tau', \vec{q}) = N_0 \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} dt\,(1 + N'(t)) \times \times \psi(\vec{q}, t + \tau') \quad .$$

This may be separated into the sum of two integrals, the first of which vanishes because of the periodicity of ψ and the second of which vanishes because of the lack of correlation between N' and ψ. Similar arguments can be made for $\rho_{\psi,N}(\tau', \vec{q})$ whence, from Eq. (A.3),

$$\rho_{\psi'}(\vec{q}, \tau') = \rho_{N'}(\tau') + N_0^2 + \rho_\psi(\tau', \vec{q}) \quad . \tag{A.5}$$

Invoking the Wiener-Khintchin theorem at $\vec{q}$, we Fourier transform Eq. (A.5) on the variable τ' and produce thereby the power spectral density of ψ', viz

$$|\underline{\psi}'(\vec{q}, \nu)|^2 = N_0^2\,\delta(\nu) + |S_N(i\nu)|^2 + |S_\psi(\vec{q}, i\nu)|^2 , \tag{A.6}$$

where $|S_N(i\nu)|^2$ is the power spectral density function of the background noise and ν is the transform parameter having dimensions of frequency. It remains to calculate $|S_\psi|^2$.

Since $\psi(\vec{q}, t)$ is periodic of period τ, it can be represented by a Fourier series of the form

$$\psi(\vec{q}, t) = \sum_n \alpha_n(\vec{q}) \exp(2\pi i n t/\tau) \quad . \tag{A.7}$$

Substitution of Eq. (A.7) in Eq. (A.2) gives

$$\rho_\psi(\vec{q}, \tau') = \sum_{n,m} \alpha_n \alpha_m^* \exp(-2\pi i m\, \tau'/\tau) \times \times \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} dt \exp[2\pi i (n-m) t/\tau]$$

$$= \sum_n |\alpha_n|^2 \exp[-2\pi i n\, \tau'/\tau] \quad . \tag{A.8}$$

The spectral density function of ψ, $|S_\psi|^2$, is obtained by Fourier transformation on τ', whence

$$|S_\psi(\vec{q}, i\nu)|^2 = \int_{-\infty}^{\infty} d\tau' \exp(2\pi i\nu\tau')\,\rho_\psi(\vec{q}, \tau')$$

$$= \sum_n |\alpha_n|^2\,\delta(\nu - n/\tau) \quad , \tag{A.9}$$

and substitution in Eq. (A.6) yields

$$|\psi'(\vec{q}, \nu)|^2 = |S_N(i\nu)|^2 + N_0^2\,\delta(\nu) + + \sum_n |\alpha_n|^2\,\delta(\nu - n/\tau) \quad . \tag{A.10}$$

We see that the ψ' power spectrum is a superposition of a continuous spectrum and a line spectrum. The continuous spectrum, arising from background noise, is to first approximation 'white' or independent of ν. The line spectrum, arising from the signal, has line intensity $|a_n|^2$. Thus, at frequency $\nu_0 = n_0/\tau$, $|S_N(i\nu_0)|^2 \cong |S_N(i\nu_{0\pm\epsilon})|^2 = \underline{\psi}'(\vec{q}, \nu_{0\pm\epsilon})|^2$. It is thus possible to correct the observed spectrum function to yield the noise-free spatially dependent intensity.

$$|\underline{\psi}'(\vec{q}, \nu)|^2 - |\underline{\psi}'(\vec{q}, \nu_{0\pm\epsilon})|^2 = |a_{n_0}|^2\,\delta\left(\nu - \frac{n_0}{\tau}\right) \quad .$$

Addendum

The authors of "Wear Rates in Automotive Engines by Liquid Scintillation Counting of Fe^{55}" (*Nuclear Science and Engineering*, **20**, 521-526 (1964)) would like to acknowledge the Special Training Division of the Oak Ridge Institute of Nuclear Studies, in whose facilities the actual wear studies were conducted. We would certainly like to thank the staff of the Division for making these facilities available for our use.

H. H. Ross
R. P. Gardner
J. W. Dunn