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Letters to the Editors

A Simple Detector for the Measurement of Room-Scattered Neutrons*

A difficulty frequently encountered in precise measurements of the critical mass or buckling of a presumably bare multiplying assembly is the effect on the assembly of neutrons which, having once escaped from it, are returned to it as a result of scattering from some massive object near the assembly (e.g., the surrounding walls of the room or the structure supporting the assembly). This effect is frequently referred to as "room return." In principle, at least, if the magnitude of the returning neutron current were known as a function of energy and of position over the entire boundary of the assembly it would be possible to include such information in the boundary conditions used to obtain the flux distributions within the assembly. The problem that confronts the experimentalist is one of discriminating between neutrons leaving the assembly and those returning to it, and establishing the relative magnitudes of these components everywhere on the boundary of the assembly.

A simple and sensitive device that makes use of the self-shielding properties of resonance detectors has been in use at Brookhaven for the past year for the purposes described above, in connection with measurements of the critical mass of bare uranium/graphite systems. These "backscatter" detectors have the property of being able to distinguish between neutrons striking them from one hemisphere and those striking from the opposite hemisphere—i.e., they exhibit what might be termed "2π-directional sense." A typical detector is illustrated in Figure 1a. It consists of a solid right cylinder of indium approximately 2 inches in diameter and 1 inch thick surrounded by a cadmium box 1/32 inch thick. At the center of each of the plane faces of the cylinder is a circular depression that accommodates two indium foils. The four foils, all of the same thickness and diameter, are labeled for convenience 1, 2, 3, and 4, from left to right. The assembly at whose boundary the relative magnitude of neutron return is to be determined is imagined to be at the left of the detector, so that foils 1 and 2 face towards it and 3 and 4 away from it. After a suitable exposure the four foils are removed and placed in a NaI(Tl) scintillation counter which records the gamma rays from the radioactive decay of In^{116}.

The activity of each of the foils may be thought of as consisting of two components, one induced by the absorption of neutrons that have come from the left and the other by neutrons that have come from the right. Let a_i be the total activity of the ith foil, ℓ_i the component induced by neutrons from the left, and r_i the component induced by neutrons from the right. Then for the ith foil

$$a_i = \ell_i + r_i \ . \tag{1}$$

Further, define the transmission factors

$$f_1 \equiv \ell_2/\ell_1 \tag{2}$$

and

$$f_2 \equiv \ell_4/\ell_1 \ . \tag{3}$$

The factor f_1 describes the transmission of neutrons through a single outer foil while the factor f_2 describes the transmission through the entire block. If the thickness of the block is very much greater than that of the foils, $r_1 = r_2$ and $\ell_3 = \ell_4$, since neutrons that have penetrated the block must have energies at which the indium capture cross section is small and for which the attenuation is therefore slow. Furthermore, if the spectral distribution of the returning neutrons is the same as that of the outgoing neutrons, $f_1 = \ell_2/\ell_1 = r_3/r_4$ and $f_2 = \ell_4/\ell_1 = r_1/r_4$. If these relations are incorporated into Eq. (1) the following four equations for the foil activities result:

$$\begin{aligned} a_1 &= \ell_1 + f_2 r_4 \\ a_2 &= f_1\ell_1 + f_2 r_4 \\ a_3 &= f_2\ell_1 + f_1 r_4 \\ a_4 &= f_2\ell_1 + r_4 \ . \end{aligned} \tag{4}$$

The chief quantity of interest is the ratio r_4/ℓ_1, i.e., the ratio of captures by foil 4 of neutrons coming from the right to captures by foil 1 of neutrons coming from the left; let this ratio be β. Then from Eqs. (4):

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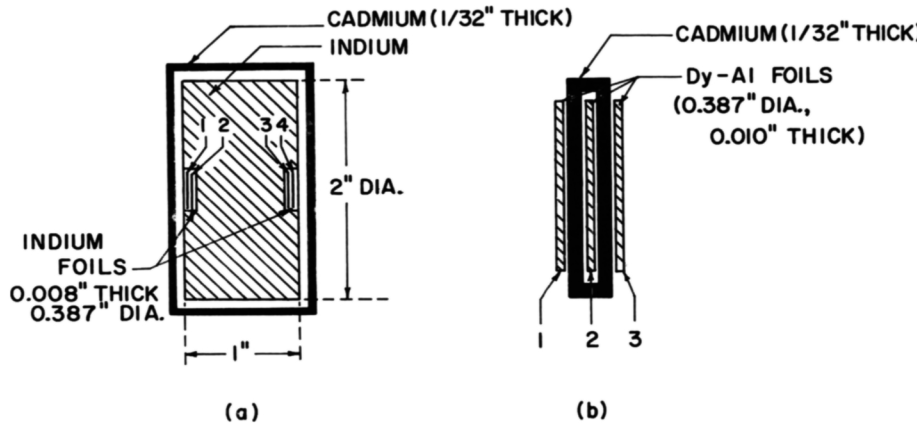

Fig. 1. Backscatter Detectors. a. Indium, b. Dysprosium.

$$\beta \equiv \frac{r_4}{\ell_1} = \frac{a_4 - a_3}{a_1 - a_2} . \quad (5)$$

The transmission factors can also be obtained from Eqs. (4):

$$f_1 = \left[\frac{(a_2/a_3) - \beta}{1 - (a_2/a_3)\beta}\right] f_2 \quad (6)$$

and

$$f_2 = \frac{1 - (a_1/a_4)\beta}{(a_1/a_4) - \beta} . \quad (7)$$

The quantity β defined in Eq. (5) is, under the assumed condition of identical spectrum for outgoing and returning neutrons, the albedo at the boundary of the assembly—i.e., the ratio of returning to outgoing neutron currents at the location of the detector. Even if the spectra are different Eq. (5) can still be used to gain qualitative insight into the return problem.

In most of our measurements the indium foils were 0.008 inch thick. The energy response of the detector depends on the thickness of the foils, it being possible in principle to obtain infinitely good resolution with infinitely thin foils. The thinner the foils, however, the greater the statistical problems of counting and, since it is usually sufficient to know the average magnitude of the return over a fairly wide energy interval, it is not necessary to use very thin foils. The energy response of the detector may be calculated according to a method given, for example, in the appendix of an article by Volpe and Klein.[1] The activity of the ith foil may be written in the form $a_i = \int a_i(E)dE$, where $a_i(E)dE$ is the activity caused by absorption of neutrons in the energy interval dE centered about E, and the integration extends upward from the energy of the cadmium cutoff. The quantity $a_i(E)$ is identical to the integrand appearing in the last equation of p. 423 of Reference (1). Using the published resonance parameters[2] of the first three resonances of In^{115} and assuming a $1/E$ spectrum, we estimate that for 0.008-inch-thick foils about 90% of the activity difference a_1-a_2 is produced by absorption of neutrons with energies between cadmium cutoff and 2 eV, and that the major part of this comes from an energy band about 0.5 eV wide centered at the first In^{115} resonance, i.e., at 1.46 eV. This energy band could be narrowed considerably by using, for instance, 0.003-inch-thick foils.

Table I presents the results of two typical measurements performed with an indium backscatter detector similar to that illustrated in Figure 1a. The foils were 0.387 inch in diameter by 0.008 inch thick. In Run No. 1 the detector was placed on a bare upper surface of the critical assembly and in Run No. 2 at a nominally flux-

TABLE I

Results of Two Typical Room-Return Measurements with the Indium Backscatter Detector Shown in Fig. 1a. Activities are in counts per minute.

	Run No. 1	Run No. 2
a_1	24,700	24,320
a_2	9,800	9,760
a_3	799	1,976
a_4	1,200	4,124
β	0.027	0.148
f_1	0.397	0.399
f_2	0.022	0.022

[1] J. J. VOLPE and D. KLEIN, "An Experimental Study of the Relative U^{235} Fission Activation as a Function of Energy in Slightly Enriched Uranium-Water Lattices," *Nucl. Sci. Eng.* **8**, 416 (1960).

[2] D. J. HUGHES and R. B. SCHWARTZ, "Neutron Cross Sections," BNL-325 (July, 1958).

symmetric position between the lower face of the assembly and the relatively massive structure that supports it. In both cases the exposure was at a critical assembly power of 7.5 watts for 45 minutes. The corresponding thermal flux at the position of the detector is estimated as $\sim 3 \times 10^5$ neutrons/cm^2-sec. Activities are given in counts per minute, corrected to the time at the end of the exposure. It will be observed from the data in the table that the neutron return at the location of Run No. 1 on the "bare" face is almost 3%, and that the return at a similar location on the supported lower face is more than five times as high. That the main source of the return at the latter location was the supporting structure and not the floor of the room (which is about 25 feet below the assembly) was established by a technique described below.

The values given in Table I for the transmission factors are approximately 0.4 and 0.02 for f_1 and f_2, respectively. The first of these agrees quite well with the results of the calculations referred to above for foils of the thickness used. The smallness of f_2 demonstrates the sensitivity of the detector, since a transmission of 2% through the block permits detection of returning currents that are as small as 1% of the outgoing current at a given location. Smaller blocks can be used (the dimensions of the one in Figure 1a were chosen so that neutrons would have to traverse a minimum path of 1 inch in indium in order to activate a foil on the opposite side) but with a resultant loss of sensitivity because of the increased transmission. With a 1/4-inch-thick indium block, for instance, we have found $f_2 \cong 0.05$.

In the case of the BNL assemblies it has been possible to correct the buckling (or reactivity) for neutron return effects without resorting to an elaborate calculation based on a complete mapping of the returning currents. As a typical example, consider the assembly in which the carbon-to-uranium-235 atom ratio was 1.15×10^4 (the fuel, highly enriched, was distributed in a nearly homogeneous manner throughout the graphite). This assembly was a rectangular parallelepiped measuring $52.5 \times 52.5 \times 72$ inches, tilted on one edge so that the four lateral faces made angles of 45° with the horizontal and the end faces were vertical. The long dimension was horizontal. The assembly was supported by a vee-shaped structure of aluminum which was in contact with the two lower faces. A measurement with the backscatter detector indicated that at a location near the center of one of the lower faces the neutron return was about 15%, whereas at the nominally flux-symmetric location on the opposite bare upper face it was only 3%. Sheets of aluminum were placed above the upper face and measurements of the backscatter fraction (i.e., β in Eq. 15) were made as a function of the thickness of the aluminum reflector, from 0 to 1.5 inches. From a plot of β versus thickness it was possible to determine the amount of aluminum for which the β at the upper face was the same as that at the lower face. The reactivity worth corresponding to a return of this magnitude into one face was then obtained by measuring the reactivity of the assembly with and without the aluminum reflector of the proper thickness on the upper face. That the thickness indicated by the backscatter detector was correct was verified by performing flux traverses in the assembly in a direction normal to the two faces in question. Without the aluminum reflector, the extrapolation distance was found to be appreciably larger at the lower face than at the upper. With the reflector in position, the two extrapolation distances were found to be equal. The total worth of the neutron return into the assembly was found in this way to be 51¢, corresponding to a change in critical buckling of about 0.1×10^{-4} cm^{-2}, the buckling itself being 13.3×10^{-4} cm^{-2}. In terms of buckling the effect was therefore a small one in this particular assembly, but in small assemblies in which leakage plays a much more important role the effect may be appreciable.

The detector described above can also be used in many cases to ascertain the source of the neutron return. To do this, one detector is placed next to the suspected source (e.g., some massive structural member) on the side towards the assembly, and a second identical one on the opposite side. The two detectors are exposed simultaneously and a comparison of the activity differences a_4-a_3 for the two then reveals whether the object is returning significant numbers of neutrons to the assembly. In this way it was found, in the case mentioned above, that the supporting structure and not the floor was responsible for most of the observed returns.

Finally, any resonance material that has suitable activation characteristics can be used in place of indium. Interpretation of the results is easier if the detector has a single dominant resonance, however. We have used gold in this way to obtain the neutron return at the large 5 eV resonance in Au^{197}. Dysprosium, which is most sensitive to thermal neutrons,[3] can be used to measure the thermal-neutron return. The dysprosium detector we have used is shown in Figure 1b. It consists of three foils of dysprosium/aluminum alloy (containing about 5% dysprosium by weight) separated from each other by cadmium. In this

[3]R. SHER, S. TASSAN, E. V. WEINSTOCK and A. HELLSTEN, "Low Energy Neutron Cross Sections of Dy^{164}," *Nucl. Sci. Eng.* 11, 369 (1961).

case the activity of foil 2 (induced by epicadmium neutrons passing through the detector in both directions) is subtracted from the activities of both foil 1 and foil 3, the resultant activity differences being caused entirely by capture of thermal neutrons. That is β(thermal) = $(a_1-a_2)/(a_3-a_2)$. This detector has the advantage of great compactness compared with the indium detector.

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An Improved Free-Surface Boundary Condition for the P-3 Approximation

One of the most common methods of solution of the monoenergetic neutron transport equation is to expand the angular dependence of the directional flux in a truncated spherical-harmonics series[1]. The resulting finite number of differential equations for the spatially dependent expansion coefficients are generally referred to as the P_N equations. N here refers to the upper limit on the summation sign of the flux expansion and thus designates the order of the expansion. Setting N equal to one yields the widely used diffusion approximation. If greater accuracy is required, the P_3 equations are often used. Only in very special instances are higher order expansions ($N > 3$) used in reactor design.

As is well known[1], in a finite-order expansion (N finite) one cannot satisfy exactly the boundary condition that, at a free surface, no neutrons return to the system, i.e., the directional flux vanishes over a hemisphere (2π) of solid angle. Thus an approximate boundary condition is required. In this letter we use the variational calculus to obtain this approximate free-surface boundary condition for the P_3 approximation. The P_1 free-surface boundary condition according to the variational method has been treated elsewhere[2] and leads to a linear extrapolation distance of 0.7071 mean free paths.

[1]B. DAVISON and J. B. SYKES, *Neutron Transport Theory*. Clarendon Press, Oxford, (1957).

[2]G. C. POMRANING and M. CLARK, JR., "The Variational Method Applied to the Monoenergetic Boltzmann Equation, Part I." *Nucl. Sci. Eng.*, 16, 147-154 (1963).

For definiteness, we consider the left-hand boundary of a slab system to be a free surface at $z = a$. The right-hand boundary is treated analogously. Slab geometry is considered for simplicity, but presumably the variational analysis could be carried through in other geometries. The P_3 directional flux expansion is

$$\phi(z,\mu) = \sum_{n=0}^{3} \left(\frac{2n+1}{2}\right) \phi_n(z) P_n(\mu), \qquad (1)$$

where $\phi(z,\mu)$ is the directional flux, z is the slab coordinate, μ is the cosine of the angle between the z axis and the velocity vector of the neutron, $P_n(\mu)$ is the n^{th} Legendre polynomial, and $\phi_n(z)$ is the n^{th} expansion coefficient, given by

$$\phi_n(z) = \int_{-1}^{1} d\mu P_n(\mu)\phi(z,\mu). \qquad (2)$$

Now, at the free surface, $z = a$, the rigourous transport-theory boundary condition is

$$\phi(a,\mu) = 0, \ (0 < \mu \leq 1). \qquad (3)$$

It is evident from the flux expansion, Eq. (1), that Eq. (3) cannot be satisfied exactly except by the trivial solution $\phi_i(a) = 0$, $0 \leq i \leq 3$. In particular, the structure of the P_3 equations demands, for a non-trivial solution, that there exist two linear relationships between the moments at the free surface. These two relationships can be written quite generally as

$$\phi_3(a) + A\phi_0(a) + B\phi_1(a) = 0, \qquad (4)$$

$$\phi_2(a) + C\phi_0(a) + D\phi_1(a) = 0. \qquad (5)$$

Mark and Marshak have each suggested methods for calculating A through D so as to approximate Eq. (3). (See reference 1 for a general discussion). Mark sets

$$\phi(a,\mu_i) = 0, \ (i = 1, 2), \qquad (6)$$

where μ_i are the two positive roots of $P_4(\mu_i) = 0$, and Marshak uses

$$\int_0^1 d\mu P_i(\mu)\phi(a,\mu) = 0, \ (i = 1, 3) \qquad (7)$$

as the boundary conditions. As argued by Davison[1] and shown by experience, Marshak's boundary conditions for the P_3 approximation, Eq. (7), are generally superior to those of Mark. (For high-order expansions, Mark's conditions may be better.) Thus, the Marshak P_3 free-surface boundary conditions are currently in general use. Using Eq. (1) in Eq. (7) and carrying out the angular integrations yields the Marshak conditions explicitly, which are, in the format of Eqs. (4) and (5),