view, the analysis makes clear the accuracies of the respective methods in question.

In addition, this conclusion would be much easier to see if angular neutron flux distributions were given in Ref. 1 as in Ref. 2 and, in particular, the values at the boundaries, which are the main source of the discrepancies. Finally, we should perhaps keep in mind the fact that not all references (Refs. 1 through 11) of Ref. 1 indeed rigorously satisfy the boundary conditions. For example, Ref. 6, which is labeled "exact," uses the discrete-ordinates method for the boundary conditions, e.g., $\Psi(t/2,\mu_j) = 0$ for $\mu_j > 0$ and $j = 1, 2, \ldots, 16$. Our method, being exact and completely analytic, corresponds to $\{\mu_j = \text{continuous}\}$ over the interval $[0,1]$.

On the other hand, while the "exact" method uses an accuracy limit of 10^{-8} in a number of interdepending iteration processes with possibilities for propagating numerical errors, we use the same accuracy limit only in two cases of matrix inversion. No approximate integrations or iterations are needed in our method.

The authors of Ref. 1 have probably observed that our eigenvalues are systematically lower than almost all calculated eigenvalues by other authors. This cannot, in our present view, be accidental: According to a theorem of the analysis of the linear operators, the lower the fundamental expectation value of a positive definite operator, the better the eigenfunction used for calculating it.

In conclusion, since our method is an exact one mathematically, both for the angular and the integrated distribution functions, the better agreement between the results of Refs. 1 and 7 should not surprise anyone because the approximate representation of the spatial dependence of the neutron fluxes is the same in both methods of these references.

C. Syros
P. Theocharopoulos

University of Patras
Laboratory of Nuclear Technology
Patras, Greece

July 11, 1979

[6]K. D. LATHROP and A. LEONARD, *Nucl. Sci. Eng.*, **22**, 115 (1965).
[7]K. H. KSCHWENDT, *Nucl. Sci. Eng.*, **44**, 423 (1971).

Reply to "The Influence of Boundary Conditions on the Precision of the Eigenvalues of the Boltzmann Equation"

The calculations of eigenvalues of the transport equation[1] that Syros and Theocharopoulos[2] refer to are based on a development of the angular neutron flux in a sphere or infinite slab in the following way:

$$\psi(x,\mu) = \frac{1}{2}\sum_{n=0}^{\infty} (2n + 1)P_n(\mu)\psi_n(x) \quad . \tag{1}$$

In our work, we solved an integral equation for $\psi_0(x)$. This equation for the flux is exact and contains the boundary condition of no incoming neutrons, facts that Syros and Theocharopoulos[2] also point out. Similarly, it is possible to derive an integral equation for the neutron current $\psi_1(x)$ and then to obtain $\psi_2(x)$ and higher order functions. In this way, one should be able to calculate the angular neutron flux in Eq. (1).

Since our work was restricted to the exact integral equation for the neutron flux $\psi_0(x)$, the angular distribution of the neutrons was not involved. Therefore, solving the integral equation by expanding $\psi_0(x)$ in Legendre polynomials in the spatial variable x should give the correct eigenvalues of the equation.

However, as pointed out by Syros and Theocharopoulos,[2,3] their eigenvalues are systematically smaller than almost all other published data. The relative deviation between their values and ours is between 1.0×10^{-4} and 1.3×10^{-3}, which is outside the estimated limits of uncertainty in our calculations. Examples for isotropic neutron scattering in an infinite slab are given in Table I. The systematic difference raises the question of whether we have had an inadequate convergence in our computations of the eigenvalues. To investigate this further, we have repeated some of our calculations with 20 terms in the series of spatial Legendre polynomials instead of 9. The results are given in Table I. It can be seen that the values obtained with 9 polynomials are in agreement with the 20 polynomial values within one unit in the 8th figure, which is the stated uncertainty.[1]

As a further check, the integral equation for the neutron current $\psi_1(x)$ has been solved with 20 polynomials in the development and for the same parameter values. As can be seen in Table I, it is not possible to get as high an accuracy as for the flux equation, but the disagreement does not start until the 8th figure.

Syros and Theocharopoulos[2] state that the fact that our results agree with those of Kschwendt[4] can be explained by a similar development of the spatial dependence. However, our results also agree with those of Kaper et al.,[5] who applied the method of Case. Some of their values are shown in Table I. They claim that the errors are less than one unit in the last decimal place, which is probably the highest accuracy obtained in a calculation of this type. The deviation between their values and ours is at most two units in the last figure given. This deviation is probably not caused by an insufficient number of terms in the development but by the limitations of our numerical procedure.

That calculations of our type do converge properly is corroborated by the work by Sanchez,[6] who used up to 150 polynomials in a similar method of solving the transport equation with linear anisotropic scattering for infinite cylinders. For the same range of anisotropy as in our work, the maximum relative deviation between his eigenvalues for 10 and 100 polynomials is 1.8×10^{-6} (see p. 90 of Ref. 6), but it is usually much less.

The evidence presented leads us to believe that our results[1] are accurate to within the error limits given. We cannot explain

[1]E. B. DAHL and N. G. SJÖSTRAND, *Nucl. Sci. Eng.*, **69**, 114 (1979).
[2]C. SYROS and P. THEOCHAROPOULOS, *Nucl. Sci. Eng.*, **73**, 108 (1980).
[3]C. SYROS and P. THEOCHAROPOULOS, *Ann. Nucl. Energy*, **4**, 495 (1977).
[4]H. KSCHWENDT, *Nucl. Sci. Eng.*, **44**, 423 (1971).
[5]H. G. KAPER, A. J. LINDEMAN, and G. K. LEAF, *Nucl. Sci. Eng.*, **54**, 94 (1974).
[6]R. SANCHEZ, "Généralisation de la méthode d'Asaoka pour le traitement d'une loi de choc linéairement anisotrope; données de référence en géométrie cylindrique," CEA-N-1831, Centre d'Etudes Nucléaires de Saclay (1975).

TABLE I

Criticality Factor c for Infinite Slab and Isotropic Scattering

Thickness (mean-free-path)	Syros and Theocharopoulos (Ref. 3)	Dahl and Sjöstrand (Ref. 1) Flux Equation (9 Polynomials)	Dahl and Sjöstrand Flux Equation (20 Polynomials)	Dahl and Sjöstrand Current Equation (20 Polynomials)	Kaper et al. (Ref. 5)
1	1.61384	1.61537 85	1.61537 854	1.61537 81	1.61537 852
2	1.27625	1.27710 18	1.27710 1823	1.27710 15	1.27710 1824
4	1.10799	1.10846 78	1.10846 78324	1.10846 76	1.10846 78323
6	---	1.05829 59	1.05829 58957	1.05829 57	1.05829 58956
8	---	1.03640 20	1.03640 20305	1.03640 19	1.03640 20303
10	1.02466	1.02487 94	1.02487 93734	1.02487 92	1.02487 93733
20	1.00702	1.00713 58	1.00713 57395	1.00713 56	1.00713 57393

why the eigenvalues of Syros and Theocharopoulos[3] are systematically lower.

E. B. Dahl
N. G. Sjöstrand

Chalmers University of Technology
Department of Reactor Physics
S-41296 Göteborg, Sweden

August 14, 1979

Comments on "Neutron-Induced Fission in a DT-Plutonium Plasma"

In two papers by Perkins,[1,2] the neutron and fusion rate enhancement by in-flight reactions created by knock-ons from fission fragment slowing down in a compressed DT-plutonium plasma has been calculated. It is found that this effect can increase the number of neutrons per fission by a factor of ~2 if the plasma temperature is near ~100 keV. This effect was predicted in a previous study by the present author,[3-5] but due to lack of research support, it was not possible to perform the tedious numerical calculations. However, no matter how important this effect might be, the higher order (but in a lower temperature range), much larger, effect resulting from the plasma heating by the fission products is completely ignored in Perkins' work. Only an analysis taking this effect into account can claim to be complete. We therefore feel the need to call the readers' attention again to the significance of this effect.

If in a high-density plasma, composed of a mixture of fissionable and fusionable material, a fission process takes place, the kinetic energy of the fission products, after being slowed down by inelastic collisions, will lead to a rise of the plasma temperature. The rate in the rise of temperature will be directly proportional to the fission energy released per unit of time if, in the temperature range, the energy density of the black body radiation aT^4 is small compared to the kinetic energy density NkT. Since the kinetic energy density is proportional to the plasma density but not the black body radiation, high plasma densities shift the range where the kinetic energy density is larger than the black body radiation energy density to higher temperatures. At the contemplated high plasma densities, the interesting temperature range is between 1 and 10 keV, where the fusion cross section averaged over a Maxwellian rises as $\langle \sigma v \rangle \simeq \text{const} \cdot T^{4.37}$. Because of this rapid rise in $\langle \sigma v \rangle$ with T, a small increase in T will greatly enhance the production of fusion neutrons. This, in turn, will accelerate the fission process. Calculating this effect, of course, implies solving the time-dependent problem, which was not done by Perkins. However, the calculation by Perkins shows that the nonthermal enhancement of fusion processes by fission product knock-ons is quite important at temperatures near ~100 keV. At this temperature, the value of $\langle \sigma v \rangle$ reaches a plateau and is therefore not very sensitive to T, and hence the rise in the fusion rate with T is here unimportant. On the other hand, according to Perkins' results, in the temperature range from 1 to 10 keV, the fusion enhancement by fission product knock-ons is not very important. It therefore follows that both calculations supplement each other, mine in the temperature range from 1 to 10 keV and Perkins' in the range near ~100 keV. In the interesting intermediate region, from 10 to 100 keV, a more complete calculation would be highly desirable. In the temperature range above 10 keV, the value of $\langle \sigma v \rangle$ does not depend on T with such a large power as in the range below 10 keV, but the knock-on effect begins to become important above 10 keV. This latter effect could be approximated by putting a larger ν value for neutron multiplication into the time-dependent analysis.

F. Winterberg

University of Nevada System
Desert Research Institute
Reno, Nevada 89506

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[1]S. T. PERKINS, *Nucl. Sci. Eng.*, **69**, 137 (1979).
[2]S. T. PERKINS, *Nucl. Sci. Eng.*, **69**, 147 (1979).
[3]F. WINTERBERG, in *Laser Interaction and Related Plasma Phenomena*, Vol. 3, p. 519, Plenum Press, New York (1973).
[4]F. WINTERBERG, *Plasma Phys.*, **15**, 71 (1975).
[5]F. WINTERBERG, *Nucl. Sci. Eng.*, **59**, 68 (1976).

Reply to "Comments on 'Neutron-Induced Fission in a DT-Plutonium Plasma' "

I was very interested in the comments presented by Winterberg.[1] Winterberg has performed calculations on the fission

[1]F. WINTERBERG, *Nucl. Sci. Eng.*, **73**, 110 (1980).