Letter to the Editor

The Particle Spectrum from a D-T Plasma

In a recent Note in this Journal, Liskien[1] used the Monte Carlo method to calculate the neutron and alpha-particle spectrum from a deuterium-tritium (D-T) plasma in thermodynamic equilibrium at fusion temperatures. We wish to point out that this problem, together with some related slowing problems, has been considered by the present author[2] using analytical methods. Assuming a general reaction of the form $A + B \to X + Y + Q$, where the mass numbers of the colliding nuclei before collision are M_1 and M_2 and those afterward are M_3 and M_4, we obtain for the energy spectrum of particles of type 3 the following expression:

$$N(E_3) = \frac{2n_1n_2}{\pi(kT)^2}\frac{(M_1+M_2)}{(2M_1M_2M_3\lambda)^{1/2}}\int_0^\infty \frac{dE_0E_0\sigma(E_0)\exp(-E_0/kT)}{(Q+E_0)^{1/2}}$$
$$\times\left[\exp\left(-\frac{(M_1+M_2)}{M_3kT}\{E_3^{1/2}-[\lambda(Q+E_0)]^{1/2}\}^2\right)\right.$$
$$\left.-\exp\left(-\frac{(M_1+M_2)}{M_3kT}\{E_3^{1/2}+[\lambda(Q+E_0)]^{1/2}\}^2\right)\right]$$
$$\mathrm{cm^{-3}\cdot s^{-1}\cdot keV^{-1}}\ , \qquad (1)$$

where $\lambda = 1 - M_3/(M_1+M_2)$ and n_1 and n_2 are the total number densities of the colliding ions in thermodynamic equilibrium at kinetic temperature T. For neutron emission, $M_3 = 1$, while for α emission, $M_3 = 4$.

It was found that a useful approximation to Eq. (1) can be written as

$$N(E_3) = \frac{n_1n_2}{\pi(kT)^2}\frac{2(M_1+M_2)}{(2M_1M_2M_3\lambda Q)^{1/2}}\int_0^\infty dE_0E_0\sigma(E_0)\exp(-E_0/kT)$$
$$\times\exp\left\{-\frac{(M_1+M_2)}{M_3kT}[E_3^{1/2}-(\lambda Q)^{1/2}]^2\right\}\mathrm{cm^{-3}\cdot s^{-1}\cdot keV^{-1}}\ . \qquad (2)$$

In both cases, $\sigma(E_0)$ is the $\sigma[M_1/(M_1+M_2)F_d]$ defined by Liskien.

It should be stressed that in arriving at Eq. (1), we have assumed that the reaction cross section is isotropic in the center-of-mass system. Our method is not restricted to this but would require further development to include terms involving the Legendre moments of the differential scattering cross section $\sigma(E_0,\theta)$. Even with the isotropic case, however, it does seem worthwhile to compare the analytical results with the Monte Carlo calculations of Liskien. In this connection I am grateful to Liskien, who has not only explained several aspects of his work to me but has most generously renormalized his calculations and supplied me with graphs comparing our results. These are shown in Figs. 1 and 2 for neutrons and alpha particles. The smooth curves are values obtained from Eq. (1), and the step curves are Monte Carlo calculations. Equation (2) is not shown here but gives reasonable agreement with the exact value except in the wings of the curves.

The agreement between the two calculations is remarkable and suggests that the effect of anisotropic scattering in the center-of-mass system is not great over the dominant regions of particle emission. It is clear that Eq. (1) can be used with some confidence for initial studies of this problem.

Finally, in Table I, we give some integral properties of the spectra which are of general interest. Simple analytical expressions for these quantities can be found in the work by Williams.[2]

M. M. R. Williams

Queen Mary College
Nuclear Engineering Department
Mile End Road
London E1 4 NS, England

August 23, 1979

[1]H. LISKIEN, *Nucl. Sci. Eng.*, **71**, 57 (1979).

[2]M. M. R. WILLIAMS, *J. Nucl. Energy*, **25**, 489 (1971).

TABLE I

Integral Properties of the Spectra*

T (keV)	$E_0 = 14.08$ MeV			$E_0 = 3.52$ MeV		
	$\Delta\bar{E}_n$	Var E_n	Rv_n	$\Delta\bar{E}_\alpha$	Var E_α	Rv_α
10	32.33	3.41×10^3	0.2417	19.33	3.52×10^3	0.9952
20	52.56	8.81×10^3	0.6233	35.64	8.88×10^3	2.4970
30	70.01	1.52×10^4	1.0738	51.25	1.52×10^3	4.2696

*Here, $\Delta\bar{E}$ is the average value minus value for zero temperature in keV; Var E is the variance $(\overline{E^2} - \bar{E}^2)^{1/2}$ in keV; and R_v is the relative variance $(\overline{E^2} - \bar{E}^2)^{1/2}/\bar{E}$.

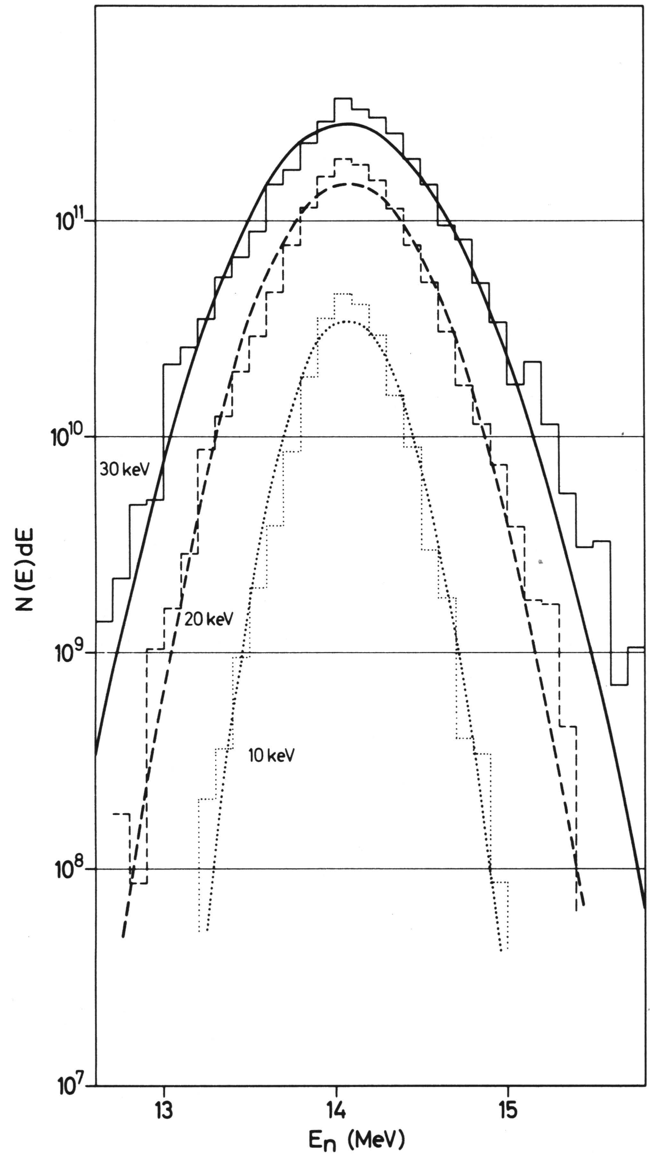

Fig. 1. The neutron spectrum at 10, 20, and 30 keV. The full lines denote the analytical solution, and the steps are the corresponding Monte Carlo results of Liskien.

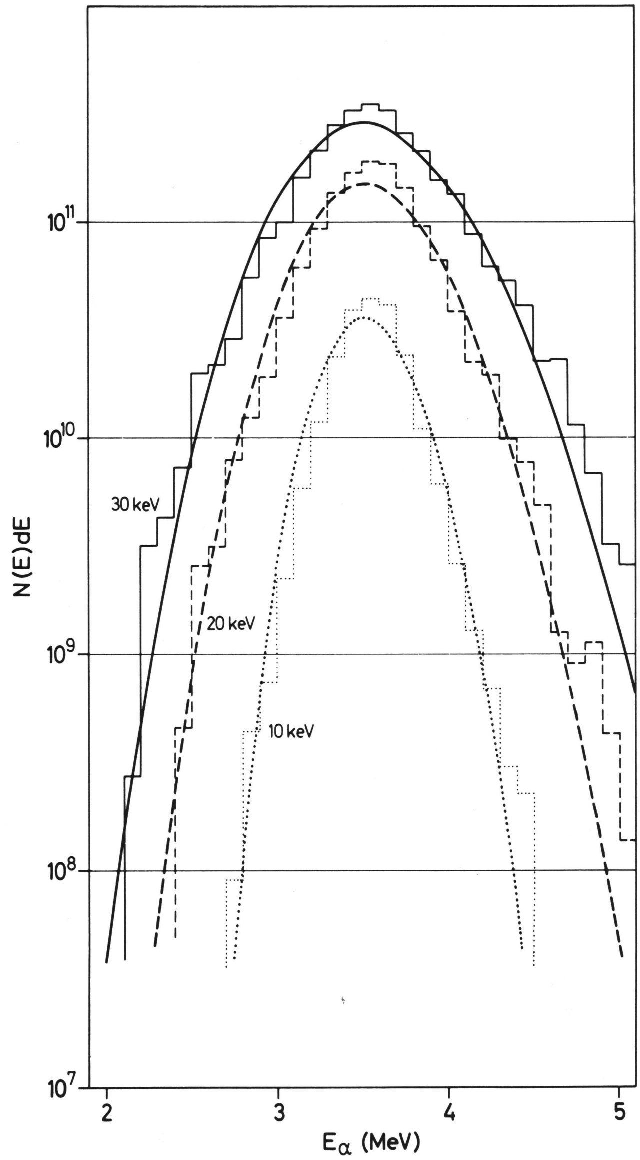

Fig. 2. The alpha-particle spectrum at 10, 20, and 30 keV. The full lines denote the analytical solutions, and the steps are the corresponding Monte Carlo results of Liskien.