# Letters to the Editor

## On the Behavior of Circulation Ratio and Algebraic Slip

In calculations of flow distribution within steam generators, an increase of circulation ratio with increasing algebraic slip has been reported. For example, in Ref. 1 a homogeneous calculation at full power produced a circulation ratio of 3.38, while a calculation for the same conditions employing an algebraic slip model using parameters developed by Lellouche and Zolotar[2] yielded a value of 3.63. Since the inclusion of interphase slip would allow the vapor phase to travel upward faster than the liquid, slip would intuitively be expected to reduce rather than raise the circulation ratio.

Though this expectation is generally correct, conditions exist for which the reverse is true, as will be shown in this Letter. This may be seen by considering a simple model of the upward flow through the evaporator of a recirculating steam generator. This flow is driven by the effective density difference between the liquid in the downcomer and the average mixture in the evaporator. Hence,

$$(\rho_l - \bar{\rho})gH = \frac{K\dot{m}^2}{2\bar{\rho}A^2} \ , \qquad (1)$$

where

$K$ = equivalent loss factor for the total evaporator flow $\dot{m}$

$\bar{\rho}$ = average density of the mixture.

Since an increase in $\bar{\rho}$ with slip tends to decrease both the driving head on the left side of Eq. (1) and the dynamic pressure loss on the right side, the behavior described below is a consequence of the relative importance of these two opposing effects. This equation may be rearranged to become

$$c(\bar{\rho}\rho_l - \bar{\rho}^2)^{1/2} = \dot{m} \ , \qquad (2)$$

where $c$ is a constant which lumps together all the constants of Eq. (1).

Since the circulation ratio, $CR$, is defined as the ratio of the total evaporator flow to the steam flow and the steam flow equals the feedwater flow in steady state (neglecting small carryover and carryunder), Eq. (2) may be rewritten as

$$\frac{c(\bar{\rho}\rho_l - \bar{\rho}^2)^{1/2}}{\dot{m}_f} = CR \ , \qquad (3)$$

where $\dot{m}_f$ is the feedwater flow rate.

Differentiating this with respect to $\bar{\rho}$, the only variable for a given set of operating conditions, we obtain

$$\frac{d(CR)}{d\bar{\rho}} = \frac{c}{\dot{m}_f} \frac{(\rho_l - 2\bar{\rho})}{2(\bar{\rho}\rho_l - \bar{\rho}^2)^{1/2}} \ . \qquad (4)$$

Thus, if $\rho_l$ is greater than twice the average density of the flow, $CR$ will increase with increasing $\bar{\rho}$. Since $\bar{\rho}$ increases with slip, $CR$ will also increase with slip under the same circumstance. However, the increase of $\bar{\rho}$ with slip will eventually cause the right side of Eq. (4) to become negative. Beyond that point, additional slip will reduce the circulation ratio. This result is illustrated in Fig. 1 (from Ref. 3), where the circulation ratio calculated by a development version of the ATHOS code is plotted as a function of assumed slip velocity. Note the denominator of Eq. (4) must be real for any circulating flow.

By way of illustration, consider the behavior of $\rho_l - 2\bar{\rho}$ at 1000 psia for a variety of slip ratios and an assumed circulation ratio of 3.5. The average flow quality was assumed to be half the exit flow quality, i.e., $\frac{1}{2} \times \frac{1}{3.5} = 0.143$. The variation of $\rho_l - 2\bar{\rho}$ with slip ratio is shown in Table I.

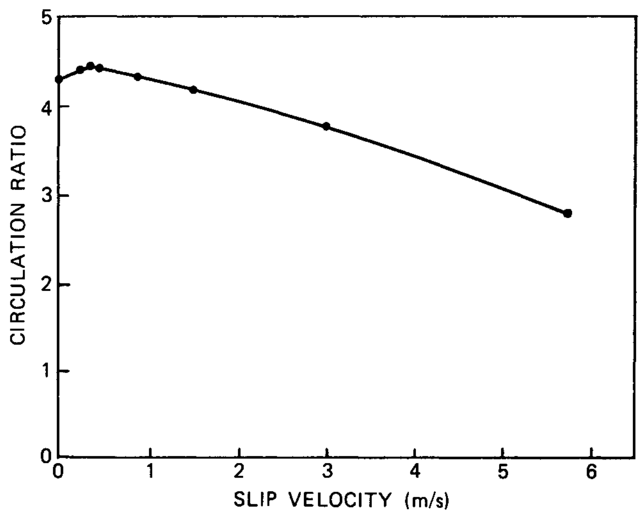


Fig. 1. Circulation ratio as a function of assumed slip velocity.

TABLE I

Variation of $\rho_l - 2\bar{\rho}$ with Slip Ratio

| | Slip Ratio | | | | | |
|---|---|---|---|---|---|---|
| | 1.0 | 1.5 | 2.0 | 3.0 | 4.0 | 5.0 |
| $\rho_l - 2\bar{\rho}$ | 22 | 15.1 | 9.44 | 0.79 | −5.5 | −10.4 |

For an average slip ratio <3.109, $\rho_l - 2\bar{\rho}$ is positive, so increasing slip increases the circulation ratio. Since the conditions assumed here are similar to those from Ref. 1 cited above, the behavior reported there is physically reasonable.

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REFERENCES

1. G. W. HOPKINS, A. Y. LEE, and O. J. MENDLER, "Thermal and Hydraulic Code Verification: ATHOS2 and Model Boiler No. 2 Data," Vol. 1, EPRI NP-2887, Electric Power Research Institute (Feb. 1983).

2. G. S. LELLOUCHE and B. A. ZOLOTAR, "A Mechanistic Model for Predicting Two-Phase Void Fraction for Water in Vertical Tubes, Channels and Rod Bundles," EPRI NP-2246-SR, Electric Power Research Institute (1982).

3. L. W. KEETON et al., "The URSULA2 Computer Program, Vol. 2: Applications (Sensitivity Studies and Demonstration Calculations)," EPRI NP-1315, Electric Power Research Institute (Jan. 1983). Used by permission.

## Richardson Extrapolation

The only aim of the present Letter is to draw the readers' attention to an old method[1] for increasing the accuracy of numerical solutions of linear problems. Customarily, new finite element (FE) or coarse-mesh (CM) algorithms are checked on well-defined benchmark problems,[2] and the reference solution of a given benchmark problem is usually obtained by a well-established program using a large number of meshes.

We are concerned here with the Richardson extrapolation,[3] which allows one to obtain higher accuracy without refining further the mesh. The method can be used independently of the geometry.

It is well known that the accuracy of a numerical solution is often proportional to some power of the mesh size $h$. The accuracy of the finite difference (FD) method is $O(h)$, that of the linear FE method is $O(h^2)$. The basic idea in Richardson extrapolation is to separate a part of the error, proportional to some power of $h$, and to eliminate it. Let us consider the problem to be solved as

$$Lu = f \quad \text{in} \quad \Omega \ , \tag{1a}$$

$$Bu = g \quad \text{on} \quad \partial\Omega \ , \tag{1b}$$

where $\partial\Omega$ is the boundary of the region $\Omega$. In numerical methods Eq. (1) is substituted by the discretized formulas

$$L_h u_h = f_h \quad \text{in} \quad \Omega_h \tag{2a}$$

$$B_h u_h = g_h \quad \text{on} \quad \partial\Omega_h \ . \tag{2b}$$

In the discretized formulas the dependence on the mesh size $h$ is indicated explicitly. The discretized form of the most frequently used operators is available in handbooks (see Ref. 4). Let us assume Eq. (2) to have a unique solution that is sufficiently smooth.[3] The solution of Eq. (2) then has the following form:

TABLE I

Richardson Extrapolation of BUG-180 Results, Using 3 and 12 Points per Hexagon, Problem GA9A1

| Number | B3[a] | B12[a] | RE[b] | B48[a] |
|---|---|---|---|---|
| 1 | 0.3798 | 0.3755 | 0.3741 | 0.3745 |
| 2 | 0.9988 | 1.0497 | 1.0667 | 1.0655 |
| 3 | 0.7781 | 0.8207 | 0.8349 | 0.8338 |
| 4 | 0.1891 | 1.2247 | 1.2366 | 1.2349 |
| 5 | 1.2358 | 1.2777 | 1.2917 | 1.2902 |
| 6 | 1.2183 | 1.2662 | 1.2821 | 1.2806 |
| 7 | 1.2450 | 1.2906 | 1.3058 | 1.3043 |
| 8 | 1.1999 | 1.2404 | 1.2539 | 1.2522 |
| 9 | 1.2147 | 1.2470 | 1.2578 | 1.2562 |
| 10 | 0.7442 | 0.7638 | 0.7703 | 0.7695 |
| 11 | 1.1766 | 1.1692 | 1.1667 | 1.1665 |
| 12 | 1.1328 | 1.1431 | 1.1465 | 1.1455 |
| 13 | 1.1938 | 1.2149 | 1.2219 | 1.2207 |
| 14 | 1.1565 | 1.1761 | 1.1826 | 1.1812 |
| 15 | 1.1522 | 1.1551 | 1.1561 | 1.1552 |
| 16 | 1.1802 | 1.1699 | 1.1665 | 1.1663 |
| 17 | 0.3494 | 0.3374 | 0.3334 | 0.3339 |
| 18 | 0.8565 | 0.8673 | 0.8709 | 0.8703 |
| 19 | 0.9093 | 0.9284 | 0.9348 | 0.9338 |
| 20 | 0.9547 | 0.9879 | 0.9990 | 0.9977 |
| 21 | 0.9665 | 1.0048 | 1.0176 | 1.0162 |
| 22 | 0.9419 | 0.9702 | 0.9796 | 0.9785 |
| 23 | 0.8859 | 0.9007 | 0.9056 | 0.9048 |
| 24 | 0.7534 | 0.7280 | 0.7195 | 0.7206 |
| 25 | 1.0347 | 0.9833 | 0.9662 | 0.9688 |
| 26 | 1.1301 | 1.1012 | 1.0916 | 1.0928 |
| 27 | 1.1038 | 1.0702 | 1.0590 | 1.0605 |
| 28 | 0.9614 | 0.9063 | 0.8879 | 0.8907 |
| 29 | 0.6444 | 0.6412 | 0.6401 | 0.6404 |
| 30 | 0.9144 | 0.8673 | 0.8576 | 0.8541 |
| 31 | 0.9867 | 0.9557 | 0.9454 | 0.9467 |
| 32 | 1.0122 | 1.0052 | 1.0029 | 1.0028 |
| 33 | 1.0913 | 1.0852 | 1.0832 | 1.0830 |
| 34 | 1.1010 | 1.0745 | 1.0657 | 1.0668 |
| 35 | 0.9778 | 0.9306 | 0.9149 | 0.9174 |
| 36 | 0.6438 | 0.6409 | 0.6399 | 0.6402 |
| 37 | 0.9128 | 0.8654 | 0.8496 | 0.8520 |
| 38 | 1.0073 | 0.9696 | 0.9570 | 0.9587 |
| 39 | 1.1131 | 1.0972 | 1.0919 | 1.0921 |
| 40 | 1.0477 | 1.0454 | 1.0446 | 1.0441 |
| 41 | 1.0215 | 1.0056 | 1.0003 | 1.0008 |
| 42 | 0.9655 | 0.9256 | 0.9123 | 0.9141 |
| Time[c] | 1.92 | 13.27 | 15.19 | 160.9 |

[a]B$n$: BUG-180 result using $n$ points per hexagon.
[b]RE: Richardson extrapolation.
[c]UNIVAC 1108 CPU min.