Letters to the Editor

Comment on a Sensitivity Coefficient in Depletion Perturbation Theory

In an earlier publication, Williams[1] developed a depletion perturbation theory for coupled neutron/nuclide fields. In that paper, a sensitivity coefficient is defined [Eq. (65)] for the variation of the initial condition of a nuclide m, as

$$S_0^m = N_0^m\{N_0^{m*} - [\Gamma_0^*\beta_0^m + P_0^*\Pi_0^m]_{\Omega,E}\} \ . \tag{1}$$

It seems that there is a contradiction between this definition and other relations as is explained later in this Letter.

The total variation in response δR takes the form

$$\delta R = \delta R_0 + \delta R_\alpha \ , \tag{2}$$

where δR_0 and δR_α represent variations in response due to perturbations of the initial concentration $\delta \mathbf{N}_0$ and of the nuclear data parameter $\delta\alpha$, respectively.

Relative variations in total and partial responses are given by

$$\frac{\delta R}{R} = \frac{\delta R_0}{R} + \frac{\delta R_\alpha}{R} \ . \tag{3}$$

Equation (64) in Ref. 1 gives δR_0 as

$$\delta R_0 = [\delta \mathbf{N}_0\{\mathbf{N}_0^* - [\Gamma_0^*\boldsymbol{\beta}_0^* + P_0^*\boldsymbol{\Pi}_0]_{\Omega,E}\}]_V \ . \tag{4}$$

Specifying the variation of the initial condition to the m nuclide, we find from Eq. (4) that

$$\delta R_0^m = [\delta N_0^m\{N_0^{m*} - [\Gamma_0^*\beta_0^m + P_0^*\Pi_0^m]_{\Omega,E}\}]_V \ . \tag{5}$$

So, a relative variation in response, due to the variation of the initial condition of the m nuclide, is given by

$$\frac{\delta R_0^m}{R} = \left[\frac{\delta N_0^m}{R}\{N_0^{m*} - [\Gamma_0^*\beta_0^m + P_0^*\Pi_0^m]_{\Omega,E}\}\right]_V \ . \tag{6}$$

We can now define a sensitivity coefficient S_0^m for the initial condition of the nuclide m by the relation

$$\frac{\delta R_0^m}{R} = \left[S_0^m \frac{\delta N_0^m}{N_0^m}\right]_V \ . \tag{7}$$

Writing Eq. (6) as

$$\frac{\delta R_0^m}{R} = \left[\frac{\delta N_0^m}{N_0^m}\cdot\frac{N_0^m}{R}\{N_0^{m*} - [\Gamma_0^*\beta_0^m + P_0^*\Pi_0^m]_{\Omega,E}\}\right]_V \tag{8}$$

and comparing Eqs. (7) and (8), we find

$$S_0^m = \frac{N_0^m}{R}\{N_0^{m*} - [\Gamma_0^*\beta_0^m + P_0^*\Pi_0^m]_{\Omega,E}\} \ . \tag{9}$$

We then conclude that the second member of Eq. (65) in Ref. 1 must be divided by the response R.

Afterward, we can easily verify that in an example calculation in Ref. 1, Eq. (93),

$$S_0 = (1 + T_f\sigma_\alpha\Phi_0) \tag{10}$$

can be obtained by substituting Eqs. (90), (91), and (92) into Eq. (9) of this Letter, and not into Eq. (65) of Ref. 1, which yields the relation:

$$S_0 = N(T_f)(1 + T_f\sigma_\alpha\Phi_0) = R(1 + T_f\sigma_\alpha\Phi_0) \ . \tag{11}$$

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REFERENCE

1. M. L. WILLIAMS, *Nucl. Sci. Eng.*, **70**, 20 (1979).

Reply to "Comment on a Sensitivity Coefficient in Depletion Perturbation Theory"

In the general field of sensitivity theory, both relative and absolute sensitivity coefficients are commonly defined, and many times both are referred to as simply "sensitivity coefficients." The choice as to which one to use depends on which one is most convenient for the particular problem of interest. Since they are proportional, it is a trivial matter to obtain one from the other. It is usually obvious which type of sensitivity coefficient is referred to by observing the content or the units. In Ref. 1, both types of expressions are used to relate the sensitivity of the response to changes in the initial condition of the nuclide field.

The coefficient of $\delta\mathbf{N}_0$ in Eq. (64) of Ref. 1 is an absolute sensitivity coefficient relating an absolute change in the initial condition ($\delta\underline{N}_0$) to the absolute change in the response (δR). This can be expressed mathematically for the m'th component of $\mathbf{N}_0$ as

$$\delta R = S_1^m \delta N^m \ ,$$

where S_1^m is the implied sensitivity coefficient in Eq. (64). Equation (64) leads to Eq. (65), which has a slightly different interpretation. Equation (65) relates the absolute response change to a relative change in the m'th component of $\mathbf{N}_0$ as

$$\delta R = S_2^m \cdot \frac{\delta N^m}{N^m} \ ,$$

where S_2^m corresponds to S_0^m in Eq. (65).

Finally, Eq. (87) [and hence Eq. (93)] gives an expression for the relative sensitivity coefficient relating the relative change in R to the relative change in the initial condition:

$$\frac{\delta R}{R} = S_3^m \frac{\delta N^m}{N^m} \ .$$

All three of the quantities S_1, S_2, and S_3 defined above are sensitivity coefficients—they are just normalized differently. The relationships between the three quantities are obviously

$$S_3^m = \frac{S_2^m}{R} = N^m \frac{S_1^m}{R} \ .$$

I believe that the point of Ref. 2 is just that Eq. (65) is an absolute sensitivity coefficient, whereas in the example calculation discussed in Ref. 1, Eq. (93) is a relative sensitivity coefficient. The assumption in Ref. 2 about this inconsistency is correct, and this author was perhaps somewhat sloppy in not identifying the differences between the two expressions. It was felt that the difference between Eqs. (65) and (84) was obvious from the context, but it evidently is not apparent to at least one reader. Equation (65) can be converted to a relative sensitivity coefficient by simply dividing by the value of the response R.

It should be emphasized that the expressions for various sensitivity coefficients in the paper are correct and that they have been verified with many numerical calculations and reactor applications. However, as with all sensitivity theory expressions, one must be careful to interpret the sensitivity coefficients correctly.

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REFERENCES

1. M. L. WILLIAMS, *Nucl. Sci. Eng.*, **70**, 20 (1979).

2. N. J. SIAKAVELLAS, *Nucl. Sci. Eng.*, **87**, 498 (1984).