

where  $\epsilon_m$  is the change in material property over the volume  $\epsilon_v$  and

$$\frac{d\phi(0)}{dx} = \phi(1+T) = 0.$$

One can easily obtain the exact solution for  $\Lambda$  as

$$\sqrt{\Lambda} \tan \sqrt{\Lambda} = \frac{-(1+\epsilon_m)}{[(1+\epsilon_m)(\epsilon_v-T) - \epsilon_v]} \quad (3)$$

A perturbation series expansion can be made for  $\Lambda$  around  $\Lambda_0$ , either in terms of  $\epsilon_m$  or in terms of  $\epsilon_v$  giving

$$\Lambda = \Lambda_0 + \partial\Lambda/\partial\epsilon_m|_{\epsilon_m=0} + O(\epsilon_m^2) \quad (4a)$$

and

$$\Lambda = \Lambda_0 + \partial\Lambda/\partial\epsilon_v|_{\epsilon_v=0} + O(\epsilon_v^2) \quad (4b)$$

The usual (classic) result using Eq. (8) of Ref. 1 and the generalized perturbation result using Eq. (11) of Ref. 1 for this problem are respectively given by

$$\delta\Lambda_u(\text{classic}) = \frac{2\Lambda_0\epsilon_v\epsilon_m}{1+T+T^2\Lambda_0} \quad (5a)$$

and

$$\delta\Lambda_g(\text{generalized}) = \frac{2\Lambda_0\epsilon_m\epsilon_v}{(1+\epsilon_m)(1+T+T^2\Lambda_0)} \quad (5b)$$

From Eqs. (3), (4), and (5), it can be easily shown that

$$\delta\Lambda_u = \epsilon_m \left. \frac{\partial\Lambda}{\partial\epsilon_m} \right|_{\epsilon_m=0} \quad (6a)$$

and

$$\delta\Lambda_g = \epsilon_v \left. \frac{\partial\Lambda}{\partial\epsilon_v} \right|_{\epsilon_v=0} \quad (6b)$$

For  $\epsilon_v = T$  and  $\epsilon_m = E$ , we get the results of Ref. 1 and Eqs. (5a) and (5b) reduce to  $(\delta\Lambda_u)$  and  $(\delta\Lambda_g)$  of Ref. 1, respectively. Thus, with reference to this specific example of Ref. 1, the classic result is accurate to first order in  $\epsilon_m$ , and the new general formulation of Ref. 1 is accurate to first order in  $\epsilon_v$ . Qualitatively, it may be possible to say<sup>2</sup> that the classic formula can be better for small  $\epsilon_m$ , and the general formula (one accurate to first order only in  $\epsilon_v$ ) can be better for small  $\epsilon_v$ . However, rigorously, the two formulations in this example are related to expansions in different parameters, and their accuracy, depending on both  $\epsilon_m$  and  $\epsilon_v$ , cannot be compared. The exact solution can be a complicated function of  $\epsilon_m$  and  $\epsilon_v$ , and the range of parameters over which one is more accurate than the other cannot be quantified.

Numerical calculations for a wider range of parameters of the perturbed problem given in Ref. 1 ( $\epsilon_m = E$ ,  $\epsilon_v = T$ ) have been made and are presented in Table I. It can be seen that our theoretical analysis satisfactorily explains the trend in the results for different values of  $E$  and  $T$ . While the general formulation is more accurate in the left region of the table, the classic formulation is better in the remaining region.

Thus, the above analysis restricted to the specific example, we hope, clearly shows that the generalized formulation of Ref. 1 has an advantage over the classic formulation only in a limited range, and the suggestion<sup>1</sup> that it replace the classic formula "as the fundamental first order diffusion theory perturbation formula" does not appear to be convincing.

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### Reply to "Generalized Perturbation Formula Versus Classic Perturbation Formula"

With regard to the recent Letter by Rao,<sup>1</sup> three points need to be made.

1. Rao apparently does not understand the newly derived (general) formula.<sup>2</sup> He states<sup>1</sup> "... the classic result is accurate to first order in  $\epsilon_m$ , and the new general formulation is accurate to first order in  $\epsilon_v$ ." This is incorrect. The correct statement is that the new general formulation is accurate to first order in  $\epsilon_m$  and  $\epsilon_v$ .

2. By a specific example, Rao shows that the classic formula is more accurate than the new formula for certain values of  $\epsilon_m$  and  $\epsilon_v$  ( $E$  and  $T$  in the example). However, an examination of the table of Rao shows that this is the case when both  $\epsilon_m$  and  $\epsilon_v$  are large, of order one or greater. Obviously any first order perturbation theory based on an expansion in a small parameter is likely to fail if this parameter is not small. In the present context, if the two formulas are misapplied, it seems to me that in any given problem there is roughly a 50/50 chance as to which one is more inaccurate.

3. Based on the numerical results referred to above, Rao questioned the suggestion<sup>2</sup> that the new formula replace the classic formula as the fundamental first order perturbation formula. I stand by this suggestion. The new formula, when properly applied (i.e., for small  $\epsilon_m$  or  $\epsilon_v$ ), clearly treats a larger class of problems. The classic formula is a special case of the new formula, restricted to perturbations first order in  $\epsilon_m$  alone. It is in this sense that the new formula should replace the classic formula.

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<sup>2</sup>J. V. MURALIDHAR RAO and S. M. LEE, *Nucl. Sci. Eng.*, **84**, 72 (1983).

<sup>1</sup>J. V. MURALIDHAR RAO, *Nucl. Sci. Eng.*, **85**, 432 (1983).

<sup>2</sup>G. C. POMRANING, *Nucl. Sci. Eng.*, **83**, 72 (1983).