TABLE I (Continued)

13.	$\int re^{-r}E_1(r)dr = rE_1(2r) - rE_1(r)e^{-r} - E_1(r)e^{-r} + E_1(2r) + \frac{1}{2}E_2(2r)$
14.	$\int rE_1^2(r)dr = \frac{1}{2}\left[r^2E_1^2(r) - 2E_1(r)e^{-r} + \frac{1}{2}e^{-2r} + 2E_1(2r) + rE_1(2r) + \frac{1}{2}E_2(2r)\right]$
15.	$\int E_2^2(r)dr = \frac{1}{3}\Big\{rE_2^2(r) - 2rE_1(r)E_2(r) + 2rE_1(r)e^{-r} - 2rE_2(2r) + 2E_1(r)e^{-r} + \frac{1}{2}\exp(-2r) + (r+6)E_1(2r) + \frac{1}{2}E_2(2r) - 2E_2^2(r) - 2E_3(2r)\Big\}$
16.	$\int r^nE_1(r)dr = \frac{1}{n+1}\left\{r^{n+1}E_1(r) + \int r^ne^{-r}dr\right\} \quad n = 1,2,3,\ldots$
17.	$\int r^nE_2(r)dr = \frac{1}{n+1}r^{n+1}E_2(r) + \frac{1}{(n+1)(n+2)}\left\{r^{n+2}E_1(r) + \int r^{n+1}e^{-r}dr\right\} \quad n = 1,2,3,\ldots$
18.	$\int \frac{E_1(r)}{r^n}dr = \frac{1}{(n-1)}r^{-(n-1)}\{E_n(r) - E_1(r)\} \quad n = 2,3,4,\ldots$
19.	$\int \frac{E_2(r)}{r}dr = E_2(r) - E_1(r)$
20.	$\int \frac{E_2(r)}{r^n}dr = \frac{1}{n-1}r^{-(n-2)}\left\{\frac{E_n(r) - E_1(r)}{n-2} - rE_2(r)\right\} \quad n = 3,4,5,\ldots$

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On the Derivation of the Equations of the Point-Reactor Model

Many authors have derived the equations for the time-dependent behavior (or kinetics) of a point reactor from diffusion and transport theory; see, for example, Refs. 1 and 2. The only aim of this letter is to present a technique of variables separation to obtain the equations of formulation of the point-reactor model from diffusion theory.

In diffusion theory, the system of equations used is (see, for example, Ref. 1)

$$\frac{\partial N}{\partial t} = Dv\nabla^2N - \Sigma_a vN + (1-\beta)k_\infty\Sigma_a vN + \Sigma_i\lambda_iC_i + S_0 \quad (1)$$

and

$$\frac{\partial C_i}{\partial t} = \beta_ik_\infty\Sigma_a vN - \lambda_iC_i \ , \quad (2)$$

where

$N(\vec{r},t)$ = neutron number density

D, v, Σ_a = diffusion constant, neutron speed, and macroscopic neutron absorption cross section, respectively

$$\beta = \Sigma_i\beta_i \ ,$$

where

β_i = delayed neutron for the i'th emitter

k_∞ = infinite-medium reproduction factor

$\lambda_i, C_i(\vec{r},t)$ = decay constant and density of the i'th type of precursor, respectively

$S_0(\vec{r},t)$ = extraneous neutron source.

All coefficients D, v, Σ_a, k_∞, β, β_i, and λ_i are constant.

We assume that

$$N(\vec{r},t) = g(\vec{r}) + n(t)f(\vec{r}) \ , \quad (3)$$

$$C_i(\vec{r},t) = p_i(\vec{r}) + c_i(t)f(\vec{r}) \ , \quad (4)$$

and

$$S_0(\vec{r},t) = q(t)f(\vec{r}) \ . \quad (5)$$

Substituting Eqs. (3), (4), and (5) into Eqs. (1) and (2) yields

$$\frac{dn}{dt} + \Sigma_a vn - (1-\beta)k_\infty\Sigma_a vn - Dvn\frac{\nabla^2f}{f} - \Sigma_i\lambda_ic_i - q = Dv\frac{\nabla^2g}{f} - \frac{\Sigma_a vg}{f} + \frac{(1-\beta)k_\infty\Sigma_a vg}{f} + \frac{\Sigma_i\lambda_ip_i}{f} \quad (6)$$

and

$$\frac{dc_i}{dt} - \beta_i k_\infty \Sigma_a v n + \lambda_i c_i = \beta_i k_\infty \Sigma_a v \frac{g}{f} - \frac{\lambda_i p_i}{f} \ . \tag{7}$$

The removal of space dependence from the left side of Eq. (6) requires that $\nabla^2 f/f$ be independent of position. This is equivalent to assuming that $f(\vec{r})$ satisfies a Helmholtz equation

$$\nabla^2 f + B^2 f = 0 \ , \tag{8}$$

where B^2 is the so-called fundamental-mode buckling. The left sides of Eqs. (6) and (7) are now independent of position and the right side of time. This is satisfied by making them all equal to zero. Then from Eqs. (6) and (7) we obtain

$$\frac{dn}{dt} = [-DB^2 - \Sigma_a + (1-\beta)k_\infty \Sigma_a] v n + \Sigma_i \lambda_i c_i + q \ , \tag{9}$$

$$Dv\nabla^2 g - \Sigma_a v g + (1-\beta) k_\infty \Sigma_a v g + \Sigma_i \lambda_i p_i = 0 \ , \tag{10}$$

$$\frac{dC_i}{dt} - \beta_i k_\infty \Sigma_a v n + \lambda_i c_i = 0 \ , \tag{11}$$

and

$$\beta_i k_\infty \Sigma_a v g - \lambda_i p_i = 0 \ . \tag{12}$$

Substituting Eq. (12) into Eq. (10), we get

$$\nabla^2 g + \frac{k_\infty - 1}{D} \Sigma_a g = 0 \ . \tag{13}$$

Then the function $g(\vec{r})$ also satisfies a Helmholtz equation. But now the fundamental-mode buckling introduced by Eq. (8) is determined by Eq. (13),

$$B^2 = \frac{k_\infty - 1}{D} \Sigma_a \ . \tag{14}$$

We introduce further symbols: the absorption lifetime l_∞, the diffusion length L, the effective reproduction factor k, the neutron lifetime l_0, the generation time l, and the reactivity ρ:

$$l_\infty = \frac{1}{v\Sigma_a} \ ; \qquad L^2 = \frac{D}{\Sigma_a} \ ;$$

$$k = \frac{k_\infty}{1 + L^2 B^2} \ ; \qquad l_0 = \frac{l_\infty}{1 + L^2 B^2} \ ;$$

$$l = \frac{l_0}{k} \ ; \qquad \rho = \frac{k-1}{k} \ .$$

Introducing these symbols into Eqs. (9) and (11), we obtain alternative formulations of the point-reactor model:

$$\frac{dn}{dt} = \frac{(1-\beta)k_\infty - (1 + L^2 B^2)}{l_\infty} n + \Sigma_i \lambda_i c_i + q \ , \tag{15a}$$

$$\frac{dn}{dt} = \frac{k - 1 - \beta k}{l_0} n + \Sigma_i \lambda_i c_i + q \ , \tag{15b}$$

$$\frac{dn}{dt} = \frac{\rho - \beta}{l} n + \Sigma_i \lambda_i c_i + q \ , \tag{15c}$$

$$\frac{dc_i}{dt} = \frac{\beta_i k}{l_0} n - \lambda_i c_i \ , \tag{15d}$$

$$\frac{dc_i}{dt} = \frac{\beta_i}{l} n - \lambda_i c_i \ . \tag{15e}$$

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