where $\bar{l} = \frac{4V_1}{S_1}$, with V_1 and S_1 being respectively the volume and bounding surface area of Region 1, and N is the concentration of absorber atoms in this region. Equation (4) is adequate for most applications and approaches the same limits as the exact expressions for specific geometries when the heated region is made very large or very small.

The following observations are made concerning the excess Doppler effect:

1. As the heated region becomes very large ($N\sigma_p\ \bar{l} \gg 1$),

$$(\Delta I)_s \rightarrow 0 \text{ since } P_{1,2} \rightarrow 0 .$$

2. As this region becomes very small

$$\left[\left(\frac{\sigma_0 \Gamma}{\Delta} + \sigma_p\right)\right] N\bar{l} \ll 1,$$

where σ_0 is the peak total cross section for the natural resonance, Γ is the total natural resonance width, and Δ is the Doppler width for the temperature of the heated region, $P_{1,2} \rightarrow 1$ over the entire resonance and $(\Delta I)_s$ assumes its maximum value.

3. For a weak resonance ($|\sigma_t' - \sigma_t| \ll \sigma_p$) or for a very small temperature change ($|\sigma_t' - \sigma_t| \ll \sigma_t$ at all energies), $(\Delta I)_s \rightarrow 0$ and $[(\Delta I)_s/(\Delta I)_v] \rightarrow 0$ regardless of the size of the heated region.

4. $(\Delta I)_s$ is always positive. This can be seen by dividing the integral of Equation (3c) into two components. One of these integrates over the center resonance region where the conditions $\sigma_t' < \sigma_t$ and $\sigma_a'/\sigma_t' < \sigma_a/\sigma_t$ are satisfied. (Note that the second inequality follows from the first for a Breit-Wigner resonance for $\sigma_p > 0$.) The second integral component covers the outer resonance regions defined by $\sigma_t' > \sigma_t$ and $\sigma_a'/\sigma_t' > \sigma_a/\sigma_t$. Since the two terms in parenthesis in Equation (3c) are either both negative or both positive and $P_{1,2}$ is positive at all energies, it follows that $(\Delta I)_s$ is always positive.

The conclusion by Khairallah and Ozeroff that the normal Doppler effect is obtained even for very small samples is based on the assumption that $|\sigma_t' - \sigma_t| \ll \sigma_t$ is generally valid (see observation 3 above). For a strong resonance (such as those responsible for the large negative Doppler coefficient in a UO_2/PuO_2-fueled fast reactor) and for a large enough temperature rise in the very small heated sample to give a measurable reactivity signal, this assumption is not correct. To evaluate the departure from the normal Doppler effect ΔI_s, ΔI_v, and the ratio $\Delta I_s/\Delta I_v$ are listed in Table I for a uranium-238 resonance at 1000 eV, assuming a temperature rise from 300 to 600 K for several diameters of a cylindrical heated region. The uranium-238 resonance parameters used are $\Gamma_\gamma = .0246$, $\Gamma_n = .0965$ (which is $\sqrt{\langle\Gamma_n^2\rangle}$ at 1000 eV); and $\sigma_p = 60$ barns corresponding to the following reactor volumetric composition: 52% sodium, 32% PuO_2-UO_2, and 16% steel; U^{238} is 77% of the total U + Pu (see Table 18 of Reference 4 for more details on the composition).

TABLE I

Diameter (cm) of Heated Region	$(\Delta I)_s$	$(\Delta I)_v$	$(\Delta I)_s/(\Delta I)_v$
0	1.18	4.22	0.28
1	0.86	4.22	0.20
5	0.36	4.22	0.085
∞	0	4.22	0

It is seen that the departure from the normal Doppler effect is significant in the limit of an infinitesimally small heated region. However, the large - region approximation, which gives the normal Doppler effect, appears adequate for regions of diameter greater than five centimeters, which probably covers most practical applications for small-sample reactivity-change measurements.

The most likely Doppler experiments with heated samples so small as to involve the above considerations are differential activation measurements using, for example, heated and unheated uranium or plutonium foils. For such experiments, the increased absorption per unit volume of heated sample reduces to

$$\Delta I = \sigma_p \int_{\text{res}} \left(\frac{\sigma_a' - \sigma_a}{\sigma_t}\right) dE \tag{5}$$

in the small sample limit. This expression differs from the normal Doppler effect since it does not involve a temperature dependence of the flux spectrum.

P. Greebler

[4] P. GREEBLER and E. GOLDMAN, "Doppler Calculations for Large Fast Ceramic Reactors - Effects of Improved Methods and Recent Cross-Section Information," GEAP-4092 (December, 1962).

Nuclear-Reactor Contribution to the Van Allen Belt

There is considerable development effort towards producing nuclear reactors for SNAP applications. This program now consists of four reactors in various stages of design and development —SNAP 10a (35 kWth), SNAP 2 (55 kWth), SNAP 8 (600 kWth) and SNAP 50 (10 - 20 MWth)[1].

From information at present available, these are all to be reflected thermal reactors intended to satisfy orbital and deep-space requirements. There is a problem peculiar to the orbital operation of high-power thermal reactors that apparently has not been considered—the decay of thermal leakage neutrons will add relativistic electrons to the Van Allen belt.[2]

The magnitude of this contribution can be readily determined from a few approximate considerations. In making such a calculation, we can assume a 15 - MWth SNAP 50 reactor in equatorial orbit for one year at 1,000 kM above Earth. Weight limitations on shielding make it reasonable to expect a thermal-neutron leakage of about 7 per cent. This means that approximately 7.5×10^{16} neutrons per second would escape.

If only those neutrons decaying within the first kM were considered, the electron contribution would be 3×10^{13} particles per second. And since the trapped electron flux at present in the Van Allen belt has an angular spread of 72° in direction of motion[3], about 0.59 of the electron contribution would be emitted in directions that would allow trapping. We conclude therefore that at least 1.5×10^{13} electrons per second would be trapped.

These particles would tend to spread throughout a 2-kM-thick donut-shape volume of space about Earth. Thus, the change in the particle density per unit time (due to reactor contribution) would be 3×10^{-11} electrons / cm^3 / sec. This would be a change in flux per unit time of 9×10^{-1} particles/ cm^2/sec^2.

The last detonation of a nuclear device in space showed that the decay time for electrons trapped in the Van Allen belt is quite long. This means that the reactor-produced electrons would effectively accumulate over the period of operation. Therefore, at the end of one year, the reactor-produced electron flux would be about 3×10^7 particles/cm^2/ sec. from one SNAP device.

This value of 3×10^7 particles/cm^2/ sec. for 0.26-MeV electrons is close to the high-solar-activity peak electron flux for Van Allen belt electrons of energy greater than 0.20 MeV.[4]

These electrons (or their Bremsstrahlung) could have significant effects on instrumentation, scientific experiments and organisms operating in that region.

In view of these considerations a thorough analysis of the problem should be performed in connection with any orbital operation of a nuclear reactor.

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Received September 17, 1963

[1]G. M. ANDERSON, *Astronautics and Aerospace Eng.* 4, P. D. 27-36 (1963).

[2]D. G. CARPENTER, ASTIA, AD 236493, p. 92.

[3]M. WALT *et al*, in *Space Research* (Kallman, Ed.), pp. 910-920. Interscience Publishers Inc., New York, (1960).

[4]J. A. VAN ALLEN and L. A. FRANK, "Radiation Measurements to 658,300 kM with Pioneer IV." State University of Iowa Research, Report 59-18, (August 1959).

Doppler Coefficient Measurements for U^{238} in Fast-Reactor Spectra

The Doppler temperature coefficient of reactivity of U^{238}, of considerable importance to large fast power breeder reactors as a prompt shutdown mechanism, has been measured in two fast spectrum assemblies on the ZPR-III reactor. This letter summarizes the experimental results (which will appear in detail in the Proceedings of the Conference on Breeding, Economics and Safety in Large Fast Power Reactors, held at Argonne National Laboratory on October 7-10, 1963, and to be published as ANL-6792).

Zoned critical methods were used on ZPR-III to provide a central zone, in which the first measurements were made; this zone had the composition and therefore the soft neutron energy spectrum of a 5,000-liter uranium monocarbide fast power breeder reactor. Since enrichment changes little with size for reactors of this size range, this measurement is pertinent to the class of large ceramic reactors. The composition, in atoms/cm^3 $\times 10^{-24}$ of U^{235}, U^{238}, C, Na, and SS were, respectively, 0.00114, 0.00732, 0.00833, 0.01061 and 0.01657. A second experiment was arranged by replacing 40% of the sodium cans of the first central zone loading with graphite. This gave a significantly increased Doppler reactivity change and provided a second check point for theoretical analysis.

The procedure of the experiment was to repeatedly change the positions of a hot Doppler element and an essentially identical cold Doppler element at the center of the reactor until the reactivity difference of the exchange was established with a precision of approximately 5×10^{-8} $\Delta k/k$. The roles of hot (800 K) and cold (300 K) elements were exchanged, and relative cold-vs-cold worths were found. The autorod-oscillation