neutron flux at times (T) subsequent to the primary neutron burst. The chopper opening has a half width of 130 μ sec. The energy spectrum of the neutrons emitted from the bottom of the reentrant hole is measured by time-of-flight over a 3-meter flight path. The neutron detector is a single 2-in. alumina-end-window BF_3 counter 6 in. long oriented with the detector axis along the neutron beam axis.

The neutron spectra for delay times (T) of 159 μ sec, 460 μ sec and 632 μ sec are shown in Figure 2; additional spectra for intermediate delay times have also been obtained. The spectra have been corrected for background and normalized to the primary-neutron-production rate; they have been corrected for neutron decay in the beryllium assembly during the time the neutrons traverse the distance from the bottom of the reentrant hole to the chopper. The neutron-detector response has been corrected by comparison with a $1/v$ detector in the same flux; the error involved in this correction may be as large as 20% for inverse velocities less than 270 μ sec/meter. The horizontal error bar gives the timing uncertainty, due mainly to the burst width of the chopper. The vertical error bars are statistical errors only and do not include systematic errors.

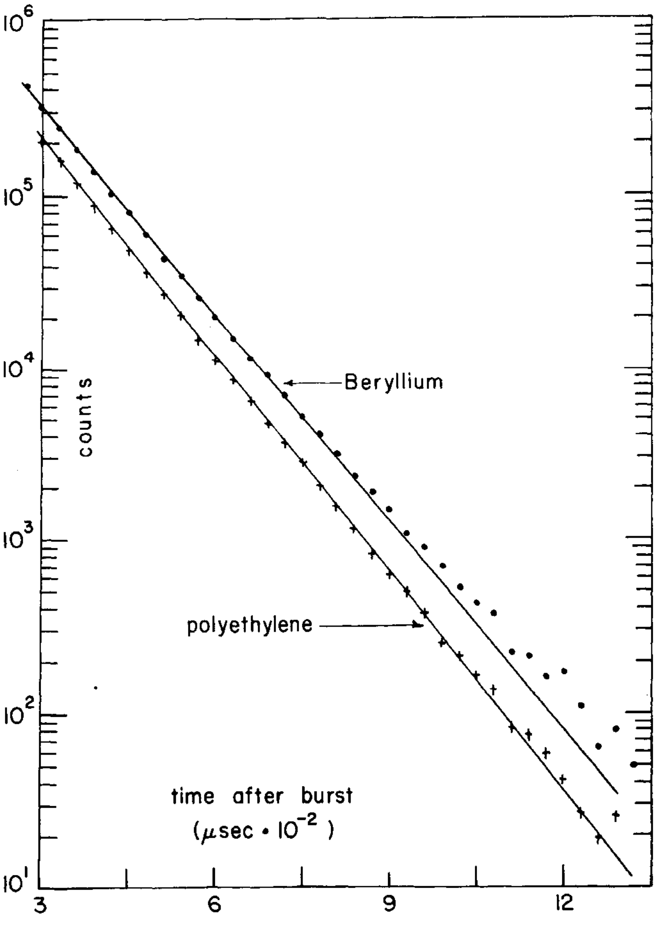

Fig. 4. Neutron die-away in beryllium and in a test assembly of polyethylene selected to give approximately the same decay time.

Three energy groups taken from these data have been replotted against time after the primary neutron burst; the curves of Figure 3 result. The energy group near the peak of the curves (0.145 - .045 eV) has been normalized to a single exponential decay. The other two groups .028 - .019 eV and 0098 - 0059 eV (which bracket the Bragg energies .022 eV and .0067 eV) indicate an initial build up and a subsequent decay approaching that of the first group.

In a different experimental set-up, using a B^{10} shield about the beryllium assembly, a conventional die-away experiment has been performed, the results of which are presented in Figure 4. Also presented are the results of a measurement performed on a polyethylene (non crystalline) assembly for the same decay time in the same environment. The polyethylene data, in contrast to the beryllium data, show no apparent change of slope.

Additional measurements are in progress on beryllium in the buckling range $B^2 = 0.1$ to $0.01/cm^2$ and will be reported upon completion.

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Expediting Danger-Coefficient Measurements by Measuring Two Samples at Once*

In danger-coefficient measurements, whether by autorod[1,2] or pile oscillator, the required

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[1] E. F. BENNETT and R. L. LONG, "Noise Measurements on a Reactor Servo-Control System", *Transactions ANS*, 5, #1, 189 (June 1962).

[2] E. F. BENNETT and R. L. LONG, "Precision Limitations in the Measurement of Small Reactivity Changes", *Nuclear Science and Engineering*, Vol. 17, (Dec. 1963).

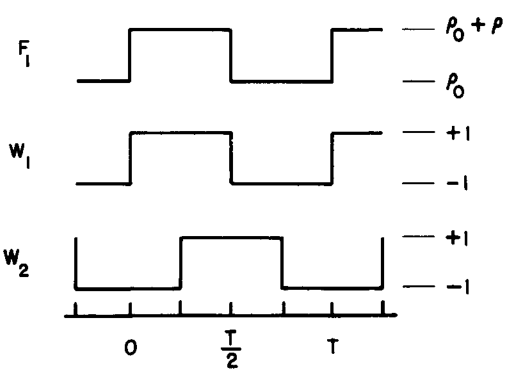

Fig. 1. Simplified reactivity profiles and weighting functions.

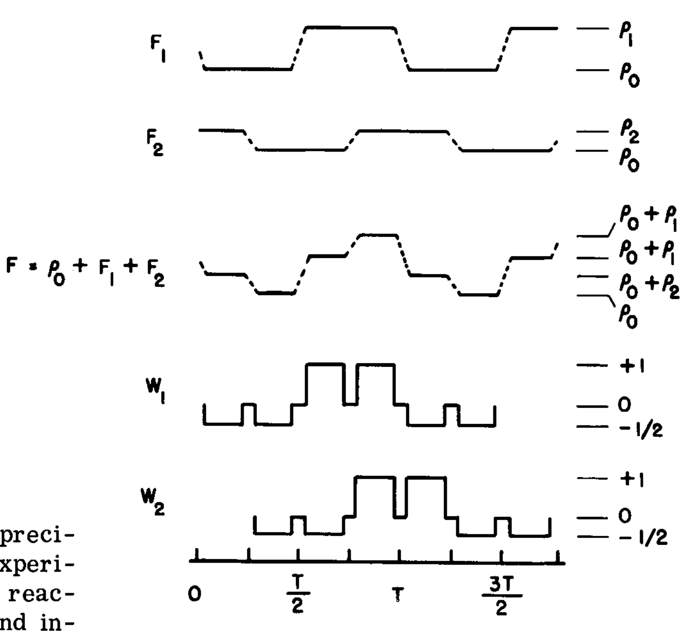

Fig. 2. Reactivity profiles and weighting functions for practical operation.

measuring time is related to the required precision by noise considerations. As a result, experiments involving measurement of very small reactivity changes are quite time-consuming and involve integrated reactor powers inconsistent with safe handling of fuel in critical assemblies. For example, the Argonne Thermal Source Reactor has been used for runs of about 10 hours at 70 watts to obtain an r.m.s. error of about 5×10^{-9} in reactivity. However, analysis indicates that it should be possible to use the measuring time more effectively and reduce fuel activation by measuring two samples at once. Although no experimental tests have been made, it is believed that, with properly adjusted equipment, this could be done without mutual interference or loss of accuracy.

Consider the three functions shown in Fig.1. Here F_1 represents the reactivity as a function of time produced by a sample of reactivity ρ which is alternately inserted into the reactor for $T/2$ seconds and removed for $T/2$ seconds, and ρ_0 is the autorod reading with the sample removed. If the reading of an autorod on such a reactor is multiplied by weighting function W_1 and integrated over one cycle, the result will be

$$\int_0^T (F_1 \times W_1)\, dt = \frac{T}{2}[(\rho_0 + \rho) - \rho_0] = \rho T/2\,, \quad (1)$$

which is a measure of the sample reactivity. However, if F_1 is multiplied by weighting function W_2 and integrated, we obtain

$$\int_0^T (F_1 \times W_2)\, dt = \frac{T}{4}[-(\rho_0 + \rho) + (\rho_0 + \rho) + \rho_0 - \rho_0] = 0 \quad (2)$$

so that no output is obtained. If a second sample were oscillated in such a way as to give a reactivity profile F_2 equivalent to F_1 delayed by $T/4$ seconds, then the integrated product of that profile with W_2 would give a result similar to Equation (1), and the integrated product with W_1 would give a zero result as in Equation (2). If both samples were oscillated, and if the autorod reading were a linear superposition of the two reactivity profiles, the integrated product of the autorod reading with W_1 and W_2 would give a result proportional to the reactivity of only one sample, with no contribution from the other. Thus the two samples could be measured independently.

The workings of this scheme can also be explained as follows: If these functions were Fourier analyzed, the results (apart from constant terms) would consist entirely of sine terms for F_1 and W_1 and entirely of cosine terms for F_2 and W_2. The possibility of separating the effects of the two samples then follows from the orthogonality of the sine and cosine functions.

In an actual situation, the signal from the autorod will contain a large noise component. As a result, an integration will in general give a non-zero result (for a run of finite duration) even if the corresponding sample is not present. However, this does not mean that the rejection of the contribution from the other sample is not perfect. In a linear system, the reactivity profiles from the two samples and the noise will be independent and will have no effect on each other. Furthermore, the contribution of noise to the result of a given integration will be the same whether the other sample is present or not.

For the autorod reading to be an accurately linear superposition of the reactivity effects of the two samples, and to avoid intermodulation effects, it would first be necessary that the autorod signal fed to the integrators be an accurately linear function of the reactivity effect of the autorod. It would further be necessary that a sample not perturb the reactivity effect of the other sample, and that neither sample perturb the effect of the autorod. In many danger-coefficient experiments, the samples are placed in a location where the flux gradient is zero to eliminate the effect of scattering. With this scheme, it would then be desired to have both samples inserted into the reactor as close to each other as possible. This would be permissible provided that the combined effect of both samples would be within the range in which the reactivity is a linear function of total absorption, i.e., in which first-order perturbation theory is valid to the desired degree of accuracy.

The functions shown in Fig. 1 would require some refinements to be useful for practical operation. The improved functions are shown in Fig. 2. Here F_1 and F_2 represent the reactivity introduced by each sample, while F is the overall reactivity. It has been pointed out[1,2] that weighting functions of the type shown as W_1 and W_2 are advantageous in rejecting extraneous reactivity drifts. An interval Δ would be provided at each sample-change time during which integration would be interrupted. This interval would have to be at least equal to the time required for changing samples plus the settling time of the autorod. Note that integration for one sample would be interrupted during the changing interval for the other sample. This would be done so that any asymmetry in the reactivity profiles of one sample for insertion and removal would not give a spurious contribution to the result for the other sample.

The results obtained from weighting functions of this type are given by

$$\begin{aligned}\int_0^{3T/2} (F \times W_1)\,dt &= \left(\frac{T}{4} - \Delta\right)\left[-\frac{\rho_0 + \rho_2}{2} - \frac{\rho_0}{2}\right.\\ &\quad \left. + (\rho_0 + \rho_1) + (\rho_0 + \rho_1 + \rho_2) - \frac{\rho_0 + \rho_2}{2} - \frac{\rho_0}{2}\right]\\ &= \left(\frac{T}{4} - \Delta\right)\rho_1 \end{aligned} \tag{3}$$

$$\begin{aligned}\int_{T/4}^{7T/4} (F \times W_2)\,dt &= \left(\frac{T}{4} - \Delta\right)\left[-\frac{\rho_0}{2} - \frac{\rho_0 + \rho_1}{2}\right.\\ &\quad \left. + (\rho_0 + \rho_1 + \rho_2) + (\rho_0 + \rho_2) - \frac{\rho_0}{2} - \frac{\rho_0 + \rho_1}{2}\right]\\ &= \left(\frac{T}{4} - \Delta\right)\rho_2 \end{aligned} \tag{4}$$

Thus each sample is measured independently of the other.

The application of this scheme to pile oscillators does not appear promising. Two oscillators would be required, running 90° out of phase. Each Fourier component of the sample reactivity profiles would be shifted in phase by the reactor transfer function. If one component of the resultant flux variations, i.e., the fundamental, were to be demodulated, the phase shift between the oscillator motion and the demodulating waveform would have to be set very accurately to avoid interference between samples. Furthermore, there would be great difficulty in avoiding spurious contributions from reactivity transients during sample travel.

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