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Letters to the Editor

On the Continuous Eigenvalue Spectrum of the Neutron-Wave Experiment

Several recent articles[1-3] have made, on occasion, misleading references to the continuous eigenvalue spectrum arising in the transport theory of the neutron-wave experiment. Much of this work can be clarified by considering plane wave solutions

$$f(x,\mu,v,t) = F(\kappa;\mu,v)\exp(-\kappa x + i\omega t)$$

to the general velocity-dependent Boltzmann equation, and then examining the associated eigenvalue problem

$$[i\omega + v\Sigma_t(v) - \kappa\mu v]F(\kappa;\mu,v) =$$
$$\tfrac{1}{2}\int_{-1}^{+1} d\mu' \int_0^\infty dv'\, v'\Sigma_s(v'\to v,\mu'\to\mu)F(\kappa;\mu',v') \equiv S[F] \quad , \qquad (1)$$

where $F(\kappa;\mu,v)$ is the eigenfunction corresponding to the eigenvalue κ. The eigenvalue spectrum of Eq. (1) for compact scattering operators S can be analyzed using the standard techniques[4] to arrive at a continuous spectrum C,

$$\kappa = \frac{i\omega}{\mu v} + \frac{\Sigma_t(v)}{\mu}\,, \quad \mu\,\varepsilon\,[-1,+1], \quad v\,\varepsilon\,[0,\infty]$$

and a point spectrum P for $\kappa \notin C$. Of course for such time-dependent problems, C becomes an area in the κ plane. If S should fail to be compact (as it does for crystalline moderators), the scattering operator may contribute an additional continuous spectrum Γ to C (see Fig. 1).

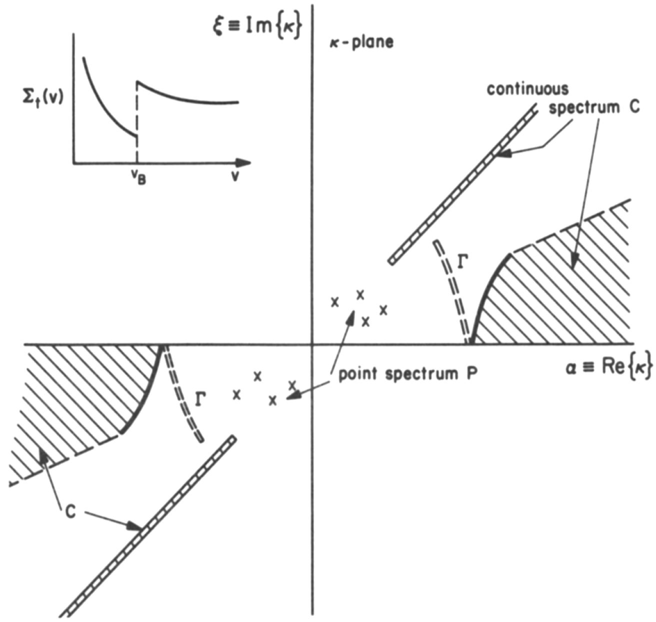

Fig. 1. Eigenvalue structure for crystalline media. Notice that Γ arises from the noncompact elastic scattering operator, while the Bragg discontinuity in the total cross-section separates the usual continuum C into disjoint domains. For noncrystalline media, only the larger areas of C would appear.

In general then, solutions to wave propagation problems can be written as a superposition of the eigenfunctions

$$f(x,\mu,v,t) = \sum_{\kappa_\ell\,\varepsilon\,P} a_\ell F(\kappa_\ell;\mu,v)\exp(-\kappa_\ell x + i\omega t)$$
$$+ \iint_C A(\kappa)F(\kappa;\mu,v)\exp(-\kappa x + i\omega t)d\kappa \quad . \qquad (2)$$

It is presumed that wave propagation experiments[3] measure the lowest discrete eigenvalue κ_0. However, detailed investigation of the point spectrum P for several simple models has suggested that for sufficiently high ω, ω^{**}, P is an empty set, and the spectral representation of the neutron density is composed entirely of the continuum eigenfunctions. Furthermore, $\kappa_0(\omega)$ may exhibit singular behavior at an even lower ω, $\omega^* < \omega^{**}$, should the point spectrum P approach C.

To investigate this change in the mathematical representation of the neutron density, the experimental data of Perez and Booth[3] have been superimposed on the κ-plane structure for graphite (using frequency-dependent coordinates for convenience—see Fig. 2). One observes that the experimentally measured dispersion curve $\kappa_0(\omega)$ intersects the continuum at a frequency $\nu^* = (\omega^*/2\pi) = 300$ cps, much lower than heretofore expected. (An interesting confirmation of this results from regarding $i\omega/v_{th}$ as an effective absorption, and then comparing its value at ω^* to some preliminary calculations[5] of $\Sigma_{a\,th}^*$ for the diffusion length experiment in graphite. Both estimates agree in the value of $\Sigma_{a\,th}^* = 0.008\ \mathrm{cm}^{-1}$.)

Of course much of the experimental data have been obtained for $\omega > \omega^*$, but this is analogous to pulsed-neutron experiments in graphite which have yielded $\lambda > \lambda^*$. Experimenters obviously appear able to measure data well past ω^* (or λ^*). It is not so obvious that these data can be subjected to the usual types of analyses; e.g., the calculation of the coefficients in a ω^{2n} or (B^{2n}) expansion.

For this reason it is desirable to study the particular case of graphite in detail. Because of the discontinuous behavior of crystalline cross sections and the fact that the elastic scattering operators S_e are not compact, one must develop rather special techniques to treat such problems. Work has been initiated on a simple model[6] which combines a separable inelastic scattering kernel with a δ-function elastic scattering term

[1]M. N. MOORE, "The Dispersion Law of a Moderator," *Nucl. Sci. Eng.*, **26**, 354 (1966).

[2]A. TRAVELLI, "The Complex Relaxation Length of Neutron Waves in Multi-Group Transport Theory," Univ. of Florida Symp. Wave, Pulse Propagation and Noise Experiments (1966).

[3]R. PEREZ and R. BOOTH, "On the Excitation of Neutron Waves," *Proc. Symp. Pulsed Neutron Research,* International Atomic Energy Agency, Karlsruhe (1965).

[4]N. CORNGOLD, "Theoretical Interpretation of Pulsed Neutron Phenomenon," *Proc. Symp. Pulsed Neutron Research,* International Atomic Energy Agency, Karlsruhe (1965).

[5]J. WOOD, "Spatial Decay Constants in the Diffusion Length Experiment," *J. Nucl. Energy, Parts A/B* (to be published).

[6]N. CORNGOLD and K. DURGUN, "Analysis of the Pulsed Neutron Experiment via a Simple Model," to be published in *Nucl. Sci. Eng.* (1967).

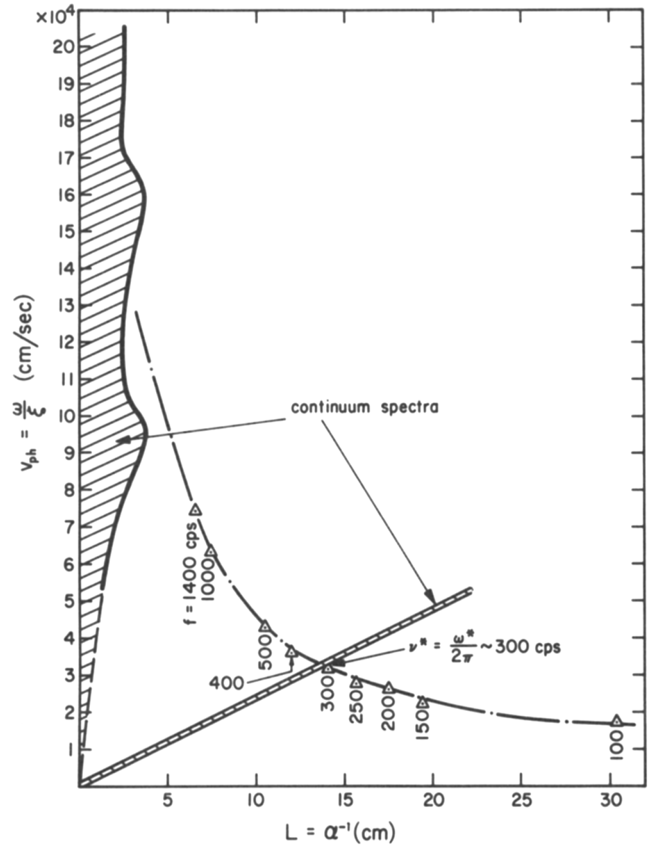

Fig. 2. Phase velocity v_{ph} vs attenuation length L for the wave experiment in graphite. The shaded regions correspond to the theoretical continuum C while the △ represent a superimposed plot of the data of Perez and Booth[3].

$$\Sigma_s(v',v) = \beta\Sigma_i(v)vM(v)\Sigma_i(v') + \Sigma_e(v)\delta(v-v')$$

to study wave propagation in graphite. The goal for such work is to provide a suitable procedure for analyzing and interpreting the experimental data obtained for $\omega > \omega^*$.

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J. J. Duderstadt

California Institute of Technology
Pasadena, California 91109

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Dr. Duderstadt brings to light in his letter some interesting points in neutron-wave theory which indeed can trap the unwary in confusion. Apparent differences in his results and mine[1] are, at first sight, puzzling. The purpose of this letter is to resolve our apparent differences.

The essential difference between our work is that our goals are different. His purpose is to map the eigenvalue spectrum of the Boltzmann equation for temporally oscillatory solutions. My purpose, considerably less ambitious, was to define an experimentally interesting region of the K plane. In this connection, my eigenvalue problem became a problem in the scalar flux (the experimentally observable quantity). For this reason, my "continuum region" is manifestly different from that of Dr. Duderstadt who (like Dr. Travelli) considers the eigenvalue problem for the vector flux. Moreover, and more important, my condition $\omega < (v\ \Sigma_T)_{\min}$ is not meant to indicate a region where 1) no continuum is found, or 2) a region where all discretum is found but rather a region in which, if a measurable dispersion law exists, one is certain to find it.

It would appear from this work, however, that the noncompactness of the crystalline moderator scattering operator can contribute the line Γ, which gives rise to relatively nonattenuated continuum solutions at lower frequencies than the minimum collision frequency. Although the amplitude of this signal may be small, one should see it in the far asymptotic region. Experimentally, the first Fourier moment[2] $(\bar{t})$ in a pulse propagation experiment should, if there is no continuum present, be a linear function of z, the detector position. If Γ is, in fact, present, one should see a linear dependence upon z for a range of z, followed by a deviation from this behavior at large z. All this should take place at $\omega < (v\ \Sigma_T)_{\min}$. Of course, the system would have to be large enough in the z direction to avoid reflections (end effects). It is to be hoped that repetitions of the Perez-Booth experiment[3] in larger systems will search for this effect.

M. N. Moore

Department of Physics
San Fernando Valley State College
Northridge, California

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[3] R. B. PEREZ and R. S. BOOTH, *Pulsed Neutron Research*, Vol. II, pp. 701-728, IAEA, Vienna (1965).