Rejoinder to J. J. Roberts, H. P. Smith, Jr., "Comments on Ash's paper 'Application of Dynamic Programming to Optimal Shutdown Control' "

The essence of the remarks concerning the above article[1], called (1) in the following, center about the equivalence of the Xenon Minimax and the time optimum (minimum time) solutions to the xenon shutdown problem. It is true that, for a given xenon upper bound override constraint $x = x_c$, the time optimal extremal is equivalent to the minimax extremal where the minimum time coincides with the allowable shutdown time of the minimax solution. However, the converse is not true. That is, minimax extremals are not necessarily time optimal extremals. For example, a valid minimax extremal could be one in which a multipulse flux train occurs (to burnout Xenon) to cause the extremal to zig-zag under the portion of the $x = x_c$ constraint line, as in Fig. 4 of (1).

The point is that the minimax criterion functional is of the terminal cost type, while the minimum time criterion functional is not. The minimax xenon depends on the terminal state (x_T, y_T), as defined in (1), only. The "cost of traversing" a minimax extremal to get to (x_T, y_T) is free; so that the "total minimax cost (including the terminal cost)" will be the same regardless of how one travels from the initial equilibrium point to the final, end of programmed shutdown, state (x_T, y_T) which lies on the desired coasting curve. This is, of course, consistent with the differential equation and xenon override constraints.

On the other hand, as mentioned, the time optimal criterion functional is not of the terminal type, but is a line integral along its extremal. The "cost of traversing" this extremal is not free—it constitutes the total cost of time optimal shutdown.

To recapitulate, the same xenon minimax is attained whether the "zig-zag" extremal or the "$x = x_c$" extremal is traversed. In general, the time optimal extremal coincides with the latter path. It all devolves to a question of which type of criterion functional to choose at the outset.

In Fig. 4 of (1), the "$62.5" curve does not take into account a xenon override constraint, but is there for comparison with the xenon constrained extremal. Nevertheless, it illustrates the fact that if this much reactivity is available, this would certainly be sufficient to keep the reactor xenon from approaching -$80 following the abrupt non-optimal shutdown, which is the amount of negative reactivity corresponding to the xenon peak at this flux level (2×10^{14} n/cm^2 sec). In Fig. 4 of (1), where the xenon constraint of $37.5 is respected, it is seen that a flux pulse train occurs (zig-zag extremal) resulting in a xenon minimax of $37.5. The time optimal extremal would result in the same minimax where the allowable shutdown time and the minimum time coincide. However, it would contain the appropriate portion of the xenon constraint line as part of its extremal arc.

With regard to relaxing the upper bound on the flux constraint, this was misinterpreted. To paraphrase the paragraph of (1) is to say that in order to make the xenon constraint line $x = x_c$ part of the extremal, $\dot{x} = 0$ and therefore $x = x_c$ hold thereon. Hence the corresponding flux u_c must satisfy the following differential equation (obtained from the Xenon-iodine state equations with $\dot{x} = 0$, $x = x_c$) along $x = x_c$,

$$[r_0 x_c - \gamma_2(w + r_0)]\dot{u}_c + [r_0 x_c - (w + r_0)]u_c = -wx_c$$

whose integral is of the form

$$u_c = A\left[\exp -\left(\frac{r_0 x_c - (w+r_0)}{r_0 x_c - \gamma_2(w+r_0)}\right) t\right] - \frac{wx_c}{r_0 x_c - (w + r_0)} .$$

If A is not chosen expeditiously, then for certain sets of the parameters u_c can pass through zero during the time in which the state is traversing the $x = x_c$ portion of the extremal. As also explained in the last paragraph of (1), if $u_c(t_2) = M$ is used to evaluate A and $M \geq 1$, the above possibility cannot occur. t_2 is the time, measured from initial shutdown, at which the extremal leaves the constraint line $x = x_c$, as in Fig. 4 of (1). For further elucidation, please be referred to my forthcoming monograph, *Optimal Shutdown Control in Nuclear Reactors*, Academic Press, Spring 1966, Chapters 2, 8 and 9.

M. Ash

E. H. Plesset Associates, Inc.
2444 Wilshire Boulevard
Santa Monica, California 90403

February 28, 1966

[1]M. ASH, "Application of Dynamic Programming to Optimal Shutdown Control," *Nucl. Sci. Eng.*, **24**, 77-86 (1966).