different diffusion coefficients (*a unified derivation*), and in this sense *all* the diffusion coefficients are on the same footing. There is no inconsistency in the derivation of D^{BC}. Further, the fact that it is double valued, depending on the cell definition, should be of no concern. Indeed, since D^{BC} gives the actual (correct to order B^2) cell leakage rate, we should be surprised if its value did not depend on the definition of the cell. Although D^{BC} is "space dependent" in this sense, it does not mean that we have not homogenized the cell, but merely that the homogenization is different for different definitions of the cell. This does *not* rule out D^{BC} as a candidate for diffusion theory calculations but does require us to state which region or cell whose reaction rates we want to preserve. It seems to be necessary to restate here that *only* D^{BC} (and the corresponding cross sections) will preserve the cell reaction rates. Rao and Lee[7] favor the use of D^{BU} because the prescription is single valued and preserves the cell reaction rates in the least-squares sense described by Köhler.[5,10] It is by no means obvious that this has any advantage over the other diffusion coefficient definitions.

In conclusion then, we must say that there is no basis at all for the rejection by Rao and Lee[7] of Benoist's corrected diffusion coefficient in favor of Benoist's uncorrected diffusion coefficient (together with the corresponding smeared cross sections). Indeed, in order to reproduce the correct (to order B^2) reaction rates within a cell deep in a lattice array, we must use the Benoist corrected prescription for that cell as already stated.[5]

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[10] P. KÖHLER, *Nucl. Sci. Eng.*, **57**, 333 (1975).

Response to "A Defense of Benoist's Corrected Diffusion Coefficient"

The arguments of Hughes[1] in favor of the Benoist corrected diffusion coefficient have been analyzed, and we stand by what we concluded in our earlier work[2] regarding this coefficient. We further clarify our stand in the following comments, which have been made with reference to Hughes's arguments.[1]

1. Hughes has stated,[1] "This eigenvalue is the same as that of the original transport equation from which Eq. (1) was derived, to order B^2. All expansions in Ref. 5 [our Ref. 3] are curtailed at order B^2, and no claim is made to reproduce the transport equation eigenvalue exactly as it seems to have been understood by Chiang and Dorning and Rao and Lee, but only to order B^2."

We point out that in Sec. II of Ref. 3, Hughes has stated, "When used with the correctly smeared cell cross sections, the resulting diffusion equation will have the same eigenvalue k^{-1}_{eff} as the original transport equation, Eq. (1)." Perhaps he implied here that the eigenvalues are the same but only to order B^2. However, our contention[2] was more fundamental in that in all the homogenization methods discussed in Refs. 2 and 3, only the asymptotic approximation to the transport eigenvalue is preserved to order B^2. To us, it is still not evident that preserving to the second order the asymptotic approximation $k_{asymptotic}$ of the transport eigenvalue k_{eff} implies preserving to the second order the transport eigenvalue itself, irrespective of the outer boundary conditions. In other words, do the coefficients of the following series,

$$k^{-1}_{asymptotic} = k^{-1}_{\infty}(1 + \epsilon k_1 + \epsilon^2 k_2 + \dots)$$

$$k^{-1}_{eff} = k^{-1}_{\infty}(1 + \epsilon k'_1 + \epsilon^2 k'_2 + \dots)$$

match irrespective of the outer boundary conditions?

2. Hughes has written[1] "Rao and Lee state that Larsen and Hughes argue that the Benoist diffusion coefficients are not suitable for use in diffusion theory calculations. This is quite untrue."

Our statement was made with reference to the following remarks made by Larsen and Hughes in Ref. 4.

a. "However, a careful look at Benoist's analysis shows that his method and the class of physical problems to which it applies differ considerably from the methods and problems considered above."

b. "The above derivation shows that Benoist's method is not one in which a homogenized diffusion equation is derived directly from a perturbation expansion of the neutron transport equation about an infinite critical medium, as in the asymptotic and buckling methods. Rather, a diffusion equation is hypothesized, and physical arguments are used to determine its coefficients. However, Gelbard has shown that the numerical value of these coefficients differs, depending on how one chooses to define a cell. In addition, no proof has been published that relates the solution of Benoist's diffusion equation to the exact solution of the transport equation."

c. "In sum, the Benoist (and related) methods do not clearly apply to near-critical systems, and their methodology, although based on a perturbation expansion, differs substantially from that of the buckling and asymptotic methods."

If the above remarks do not imply that the Benoist diffusion coefficients are not suitable for diffusion theory calculations approximating the original transport problem, and are not on the same footing as the Larsen or the Deniz-Gelbard diffusion coefficients, then we do not understand the need for the above referred comments in a lengthy discussion on Benoist's method under Sec. VII.F of Ref. 4. It was these remarks that led us to attempt to derive the Benoist diffusion equation using the asymptotic method, and we found this to be possible[2] for the case of the Benoist uncorrected diffusion coefficient D^{BU} but not for the Benoist corrected diffusion coefficient D^{BC}.

3. Hughes has further stated,[1] "Rao and Lee themselves show how the asymptotic method can be used to produce one

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[1] R. P. HUGHES, *Nucl. Sci. Eng.*, **83**, 408 (1983).

[2] J. V. MURALIDHAR RAO and S. M. LEE, *Nucl. Sci. Eng.*, **82**, 71 (1982).

[3] R. P. HUGHES, *Nucl. Sci. Eng.*, **67**, 85 (1978).

[4] E. W. LARSEN and R. P. HUGHES, *Nucl. Sci., Eng.*, **73**, 274 (1980).

equation of the form (2) for the Benoist uncorrected diffusion coefficient D^{BU}, which is entirely equivalent to Eq. (8) of Ref. 5," [our Ref. 3].

He has also stated,[1] "..., Ref. 5 [our Ref. 3] shows quite clearly that the macroscopic variation is included in the derivation of *all* the different diffusion coefficients (*a unified derivation*), and in this sense *all* the diffusion coefficients are on the same footing. There is no inconsistency in the derivation of D^{BC}." And, "... there is no basis at all for the rejection by Rao and Lee of Benoist's corrected diffusion coefficient in favor of Benoist's uncorrected diffusion coefficient.... "

In the above statements Hughes implies that Eqs. (1) and (2) of Ref. 1 are identical. Our main point in Sec. IV of Ref. 2 was to show that they are not identical. In Eq. (1) of Ref. 1 the macroscopic variation of flux has gone into the determination of the homogenized constants; everything is known and there is nothing further to be determined. Whereas, Eq. (2) of Ref. 1 is a differential diffusion equation to be solved to obtain the macroscopic variation $A(x)$. By taking $A \equiv \exp(iBx)$ we can reproduce Eq. (1) from Eq. (2). But can we reproduce Eq. (2) from Eq. (1)? While D^L, D^{DG}, and D^{BU} have been shown to satisfy both Eqs. (1) and (2) of Ref. 1, D^{BC} has been shown to satisfy only Eq. (1) of Ref. 1, and it is not obvious to us that D^{BC} obeys Eq. (2) of Ref. 1.

While attempting to derive the diffusion equation for D^{BU}, we found that a differential diffusion equation could be derived only by using an important consequence of the asymptotic method, namely, the independence of the slow and fast variables, which implied that there was no variation in the macroscopic flux $A(x)$ when calculating averages over a cell, as shown in Eq. (18) of Ref. 2. Since the definition of D^{BC} does require the variation of the macroscopic flux over the cell, we feel it to be inconsistent with the procedure used to derive differential diffusion equations for D^L, D^{DG}, and D^{BU}. Even if Eq. (41) of Ref. 2 is written as

$$-D^{BC}(x_0)\frac{d^2A}{dx^2}\bigg|_{x_0} + \Sigma_a^{BC}(x_0)A(x_0) = \frac{\nu\Sigma_f^{BC}(x_0)}{k}A(x_0) \ ,$$

does it imply that the following differential equation exists

$$-D^{BC}(x_0)\frac{d^2A(x)}{dx^2} + \Sigma_a^{BC}(x_0)A(x) = \frac{\nu\Sigma_f^{BC}(x_0)}{k}A(x) \ ?$$

In the derivation of the last equation for the cases of D^L, D^{DG}, and D^{BU}, it is required that $A(x)$ should not vary in calculating cell averages, and this is inconsistent with the definition of D^{BC}. This was the basis for our statement that D^{BC} is not a candidate for diffusion theory calculations.

We do not agree with Hughes's statement that macroscopic variation is included in the derivation of all the different diffusion coefficients.[2,4] Inclusion of the macroscopic variation of the flux is essential only for the case of D^{BC}. We feel that the aim of homogenization should be to get the constants using the fine structure of the distribution in an approximate way (asymptotic) so that they can be used in the diffusion equation to obtain the macroscopic variation $A(x)$. In other words, the homogenized constants define $A(x)$ and not vice versa. Of course in reality, the fine structure in a cell depends on its position in the finite global system, but the correction for this can be obtained only by solving the original transport equation, Eq. (1) of Ref. 3 with appropriate boundary conditions. Both in the buckling and in the asymptotic methods, the dependence of fine structure on cell position is ignored. This is clear from Eqs. (2) and (4) of Ref. 3 and in the requirement of independence of slow and fast variables in Ref. 4.

4. Lastly, Hughes has stated, "Rao and Lee favor the use of D^{BU} because the prescription is single valued and preserves the cell reaction rates in the least-squares sense described by Köhler. It is by no means obvious that this has any advantage over the other diffusion coefficient definitions."

The reasons for our favoring D^{BU} have already been stated in Sec. V of Ref. 2. Further arguments in favor of D^{BU} can be found in Ref. 5.

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[5] V. DENIZ, "A New Consistent Definition of the Homogenized Diffusion Coefficient of a Lattice, Limitations of the Homogenization Concept, and Discussion of Previously Defined Coefficients," *Proc. Specialists' Mtg. Homogenization Methods in Reactor Physics*, Lugano, Switzerland, November 1978, IAEA-TECDOC-231, p. 521, International Atomic Energy Agency (1980).