sampling instants[3]. These hidden oscillations can occur if the open-loop transfer function of the corresponding continuous system has complex poles. (The magnitude of the imaginary parts of the complex roots determines the sampling rate required to avoid hidden oscillations.) In this case the transfer function apparently does have such complex roots, and the criterion is necessary in order to see the oscillations in the output.

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[3] E. I. JURY, "Hidden Oscillations in Sampled Data Control Systems," AIEE Trans., Pt. II, **76**, 391 (1956).

Reply to T. J. Marciniak's Comments on "Stability Analysis of a Sampled-Data Controlled Nuclear Reactor System"

The following remarks are in reply to the preceding Letter. Figures 1, 2, and 3 in the paper by Reisch[1] help in deriving Fig. 4 which is the usual form of a block diagram representing a sampled-data system. The sampler of Fig. 1 is identical with the sampler of Fig. 4. The comparator, which is connected to the scanning device and the memory unit, selects and places in the memory the highest of the two values: the temperature stored in the memory or the temperature of the cooling channel which is being scanned at that moment. In this way there is always only one value in the memory which, at the end of the checking cycle, equals the highest channel temperature during the cycle. This method does imply a variable time delay which can be handled by the modified z-transform method. Nevertheless, I preferred not to use it because my purpose was to demonstrate the usefulness of the application of modern control theories for reactor dynamic studies, and I think that the explanation and use of the modified z-transform technique would have extended the paper unnecessarily. Instead, I proposed the use of a scanning device sufficiently fast so that the variation of this time delay can be neglected.

To fully explain in one page "Some Basic Properties of the z-Transform Theory" is naturally impossible. I omitted both negative times and frequencies when deriving Eq. (12). It is stated properly that $x(z)$ is an infinite series, and it is left to the reader to understand that the z-transform tables mentioned contain these series in closed form.

My choice of the stability criterion in the z plane is deliberate. The roots of the characteristic equation can be calculated with sufficient accuracy by using double precision calculations which are available, according to my knowledge, in all types of modern computers and which are laborious to use. The coefficients of the characteristic equation are used even when the step responses of the sampled system are calculated in the time domain. There is no graphical procedure at all; both the roots of the characteristic equations and the time responses are calculated in the same run from the system parameters.

It is stated properly that the power series expansion method for inverse z transformation is sometimes called long division, i.e., there are two names for a single method. I agree that it would have been useful to point out that z-transform inversion is not unique and therefore must be handled with care.

The choice of the sampling frequency is a delicate feature. To be mathematically exact one should use a sampling frequency at least twice as high as the highest frequency of the continuous system. The fact is that the overwhelming parts of the existing systems have no frequency limit above which the system amplification disappears. Nevertheless the z-transform technique is widely used, for example, in the discussed paper. As high sampling frequencies are economically penalized, the problem is to find a formula for the lowest permissible sampling frequency without running the risk of hidden oscillations. My choice is somewhat arbitrary but I think it can be justified with a control technique background.

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January 17, 1967

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[1] F. Reisch, *Nucl. Sci. Eng.*, **26**, 378 (1966).