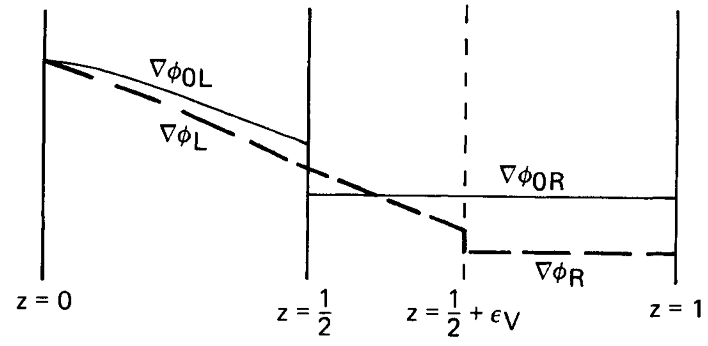


Fig. 1.

and

$$\nabla\phi_R = \nabla\phi_{0R} + O(\epsilon_M) \qquad \text{in } \frac{1}{2} + \epsilon_V \leqslant z \leqslant 1 \ . \tag{9}$$

Hence, if the interface shift perturbation is treated as a material perturbation, then, as justified by our Eq. (8) and in fact demanded by the rigorous derivation of the classical first-order perturbation formula, $\nabla\phi$ should be replaced by $\nabla\phi_{0R}$ in the integral over the perturbed volume. If this is done, the exact Eq. (15) of Ref. 1 will reduce to the classic first-order perturbation formula, Eq. (6) of Ref. 1, which will give results correct to the first order in $\epsilon_M$.

Parenthetically, we remark that the requirement of current continuity across the interface causes a discontinuity in $\nabla\phi_0$ at $z = \frac{1}{2}$. However, we see that

$$\nabla\phi_{0R}\left(z = \frac{1}{2}\right) = \frac{D_L}{D_R}\nabla\phi_{0L}\left(z = \frac{1}{2}\right) = \left(1 + \epsilon_M \frac{\delta D}{D_R}\right)\nabla\phi_{0L}\left(z = \frac{1}{2}\right)$$
$$= \nabla\phi_{0L}\left(z = \frac{1}{2}\right) + O(\epsilon_M) \ . \tag{10}$$

Thus, this discontinuity is of order $\epsilon_M$ only, and this cannot cause any inconsistency in our Eq. (8).

For the corrected perturbation formula, we see from Fig. 1 that, by Taylor series expansion,

$$\nabla\phi_L = \nabla\phi_L\left(z = \frac{1}{2}\right) + O(\epsilon_V) = \nabla\phi_{0L}\left(z = \frac{1}{2}\right) + O(\epsilon_V)$$
$$\text{in } \frac{1}{2} \leqslant z \leqslant \frac{1}{2} + \epsilon_V \ . \tag{11}$$

Further, as also noted by Rahnema and Pomraning,

$$\nabla\phi_L \neq \nabla\phi_{0R} + O(\epsilon_V) \quad \text{in } \frac{1}{2} \leqslant z \leqslant \frac{1}{2} + \epsilon_V \ . \tag{12}$$

Thus, in the region of perturbation, $\nabla\phi$ ($\equiv \nabla\phi_L$) only approaches $\nabla\phi_{0L}(z = \frac{1}{2})$ and not $\nabla\phi_{0R}(z = \frac{1}{2})$ as $\epsilon_V \to 0$. If the interface shift perturbation is treated as a volume perturbation (of order $\epsilon_V$), then the exact Eq. (15) of Ref. 1 should be converted to a surface integral over the unperturbed surface and $\nabla\phi$ replaced by $\nabla\phi_{0L}(z = \frac{1}{2})$ to get the corrected perturbation formula, Eq. (17) of Ref. 1, which will give results correct to the first order in $\epsilon_V$.

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## Reply to "On the Use of First-Order Perturbation Theory in Interface Shift Problems"

Rao and Lee[1] seem to have attributed more to our Note[2] than we intended. We did not, as these authors seem to imply, claim that the classic derivation of the standard first-order perturbation formula is incorrect. Indeed, the traditional derivation, equation, and interpretation of classic first-order perturbation theory are all entirely correct if the perturbed problem differs from the unperturbed one by order $\epsilon_M$ (presumed to be small). Our only purpose was to point out that this classic result is, in fact, *limited* to perturbations of order $\epsilon_M$. More specifically, the classical first-order perturbation formula will not correctly treat the case of a slight internal interface shift, characterized as an order $\epsilon_V$ perturbation. This is so in spite of the fact that $\phi_0$ and $\phi$ differ by a small amount, of order $\epsilon_V$, for this class of problems.

It appears that all of the arguments and the analysis of Rao and Lee[1] do no more than repeat the arguments we have made,[2] in a somewhat different (and to us, more confusing) language. It might also be useful in this interchange to point out that a quite general perturbation formula, correct to first order in both $\epsilon_M$ and $\epsilon_V$, has recently been obtained.[3] This new formula reduces to the classical first-order perturbation formula for perturbations of order $\epsilon_M$ to Eq. (17) of Ref. 2 for the order $\epsilon_V$ interface shift problem and, in general, correctly treats an arbitrary perturbation in first order, which alters the scalar flux and current by an order $\epsilon$ amount.

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[1]J. V. MURALIDHAR RAO and S. M. LEE, *Nucl. Sci. Eng.*, **84**, 72 (1983).
[2]F. RAHNEMA and G. C. POMRANING, *Nucl. Sci. Eng.*, **78**, 393 (1981).
[3]G. C. POMRANING, *Nucl. Sci. Eng.*, **83**, 72 (1983).