The details of this calculation are presented elsewhere (*6*).

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Failure of Neutron Transport Approximations in Small Cells in Cylindrical Geometry

A common problem in reactor design is the calculation of the thermal-neutron flux fine structure in fuel element cells. When the cell dimensions become less than a mean free path, it has been popular to use P_n and S_n approximations to the Boltzman equation, rather than diffusion theory, P_1. However, it has been found that in cylindrical geometry, approximations such as P_3, S_4, and S_8, can in some instances be poorer than P_1. This is not in contradiction to many successful comparisons of P_3 with experiment. The latter have typically been loosely packed, room-temperature lattices; the cases studied here are tightly packed lattices with a low-density coolant, as might be encountered in some power reactors.

TABLE I

Lattice Characteristics

Lattice No.	Spacing, a, (in cm)	Void fraction, γ	H_2O cross sections, cm^{-1}	
			$\Sigma_a{}^1$	$\Sigma_{tr}{}^1$
1	1.524	0.25	0.0088	1.053
2		0.5	0.00587	0.702
3	1.143	0.25	0.0088	1.053
4		0.5	0.00587	0.702
5		0.75	0.00293	0.351
6		0.95	0.000587	0.0702

Table I gives characteristics of lattices arbitrarily chosen for study. These are all 1.5% enriched uranium metal rods, 0.3 in. in diameter, and separated by a distance, a, in a square lattice. In all cases $\Sigma_{a^0} = 0.387\ cm^{-1}$ and $\Sigma_{tr^0} = 0.393\ cm^{-1}$ for the fuel. Boiling light water with a 600 psig density, 0.8 $(1 - \alpha)$, is assumed to be the coolant. α is the void fraction. Lattices near the bottom of the table are unlikely in practical reactors, but are useful in this study. The choice of a Maxwellian spectrum at 0.045 ev, for all lattices, simplifies the study and does not affect the conclusions.

Methods used to compute the fine structure in these cells having a constant source in the coolant are as follows:

1. Monte Carlo (*1*). This is two dimensional, (xy), and it is necessary to approximate the fuel rod as a square with the corners removed.
2. Amouyal's approximation (*2*). This uses cylindrical geometry diffusion theory in the coolant and transport theory in the fuel.
3. Diffusion theory in cylindrical geometry (which gives the same result as diffusion theory in the equivalent one-dimensional slab geometry).
4. P_3 in cylindrical geometry.
5. S_8 in cylindrical geometry.
6. S_4 in cylindrical geometry.
7. S_8 using an equivalent one dimensional slab geometry.
8. P_3 using an equivalent one dimensional slab geometry.

Regarding the S_n methods, these exist in two forms, the so-called SNG and the DSN (*3*) and both are examined here.

In methods having an outer cell boundary which is cylindrical, the cell radius, r_1, as obtained from

$$\pi r_1^2 = a^2 \tag{1}$$

To obtain the half-thicknesses of the equivalent fuel and coolant slabs, δ_0 and δ_1, from the fuel and cell radii, r_0 and r_1, the criteria

$$\bar{\varphi}_{\text{fuel}}{:}\phi_{\text{fuel edge}} = C_1;\ \phi_{\text{fuel edge}}{:}\bar{\varphi}_{\text{coolant}} = C_2 \tag{2}$$

can be applied to cylindrical and slab geometries using diffusion theory, requiring that C_1 and C_2 be the same in both geometries, with the approximate results:

$$\delta_0 \cong \sqrt{\frac{3}{8}}\, r_0 \tag{3}$$

$$\delta_1 \cong \frac{\sqrt{6}\, r_0}{1 - \left(\frac{r_0}{r_1}\right)^2} \left\{ \frac{ln\, \frac{r_0}{r_1}}{1 - \left(\frac{r_0}{r_1}\right)^2} - \frac{3}{4} + \frac{1}{4}\left(\frac{r_0}{r_1}\right)^2 \right\} \tag{4}$$

Tables II and III give the results of these methods for selected lattices from Table I. The P_1 results in Table III are the same as those of Table II by virtue of the definition of slab equivalence. The meaning of the error assigned to the monte carlo results is that the band defined has a 95% chance of encompassing the true answer. If monte carlo is regarded as correct the following inferences can be made.

(*a*) While possibly for larger lattice sizes (in terms of

TABLE II

RESULTS FOR THE DISADVANTAGE FACTOR, $\bar{\varphi}_1/\bar{\varphi}_0$, IN CYLINDRICAL GEOMETRY

Lattice No.	$\Sigma_{tr}^1 (r_1 - r_0)$	Disadvantage factor						
		Monte Carlo (xy geometry)	Amouyal	P_1	P_3	DSN-S_8	SNG-S_8	SNG-S_4
1	0.504	1.135 ± 0.031	1.170	1.051	1.165	1.242	1.233	1.214
2	0.336		1.169	1.039	1.188	1.302	1.289	1.261
3	0.278		1.155	1.036	1.207	1.291	1.276	1.276
4	0.185	1.137 ± 0.012	1.159	1.030	1.265	1.386	1.364	1.373
5	0.0926		1.169	1.023	1.440		1.625	1.666
6	0.0185	1.161 ± 0.010	1.186	1.018				
Typical IBM-704 running time, min		40	(hand)	(hand)	3	4	4	3

TABLE III

RESULTS FOR THE DISADVANTAGE FACTOR, $\bar{\varphi}_1/\bar{\varphi}_0$, IN SLAB GEOMETRY

Lattice No.	$\Sigma_{tr}^1 \delta_1$	Disadvantage factor			
		DSN-S_8	SNG-S_8	P_3	P_1
1	.382	1.173	1.178	1.098	1.051
2	.254	1.155	1.162	1.077	1.039
3	.220	1.150	1.160	1.072	1.036
4	.147	1.138	1.160	1.059	1.030
5	.0734	1.121	1.186	1.046	1.023
6	.0147	1.101	(program failed)		1.018
Typical IBM-704 running time, min		3	3	1	(hand)

mean free paths) the P_3, S_4, and S_8 in cylindrical geometry are more nearly correct than diffusion theory, these approximations are worse than diffusion theory for small lattices.

(*b*) Amouyal's approximation and the use of DSN-S_8 in an equivalent slab geometry are surprisingly close to monte carlo results over the entire range studied.

(*c*) The DSN is to be preferred over the SNG, at least in slab geometry.

(*d*) In the limit of a small $\Sigma_{tr}\delta_1$, the three reliable slab geometry methods (DSN, P_3, and P_1) apparently converge on finite disadvantage factors equal to the max to average flux in the fuel. Monte Carlo also does this in xy geometry. However, in cylindrical geometry only the P_1 and Amouyal approximation seem to give a reasonable result in this limit.

These results are also supported by a number of other calculations not reported here.

While it is not the purpose of this brief note to attempt an analysis of these effects, some comments might be made. It is suspected that the anomalously large values of the disadvantage factor in higher order approximations in cylindrical geometry may be a consequence of the boundary conditions at the cell boundary. Physically, these cell boundaries are flat, and the curvature artificially introduced may be a source of difficulty. Slab and xy geometries have no such artificiality. It might be reasoned that an $r\theta$ cylindrical geometry P_3, S_4, or S_8 could give reasonable results, if the outer boundary conditions would be those applied across flat faces at the true cell boundary, *à la* slab geometry, or xy geometry.

It is concluded that the use of P_3, S_4, or S_8 in cylindrical geometry may not always be the best approximation, especially if the mean free paths somewhat exceed the lattice sizes. Occasional comparison with better behaved methods such as Monte Carlo, Amouyal's approximation, and equivalent slabs, may be necessary in individual cases.

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