lethargy, u, is given by $u = \log 10/E$ with E the energy in Mev. The highest energy group employed had an upper bound of 14.9 Mev. The tabulation of the fission spectrum (*3*) given in reference *4* was employed. Oxygen cross sections and anisotropic scattering data were taken from reference *5*. Legendre polynomial fits to the available data for deuterium (*6–10*) were employed to express the anisotropic scattering data in the same form as that for oxygen in reference *5*, with weight given to what appeared to be the best data (*6*, *7*, *10*). The calculation gave the results that, per fission neutron, there are 0.00414 $(n, 2n)$ reactions with deuterium, 0.00472 (n, α) and (n, p) reactions [mainly (n, α)] with oxygen, and 0.00097 inelastic collisions with oxygen. The net loss of neutrons per fission neutron is thus 0.00058. The $(n, 2n)$ effect is somewhat larger than stated by Weinberg and Wigner (*11*), but much smaller than would be calculated from the cross sections assumed by Howerton (*12*) or Emmerich (*13*).

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Formulas for Thermal Reactors with Some Resonance Fission

The contribution of fissions induced by epithermal neutrons to the reactivity of primarily thermal reactors is of interest in reactors having low moderator to fuel ratios. In treating this effect it has been common to use the following definitions:

$$(k_{\text{thermal}}) \qquad k_t = \epsilon\eta_t f p_{28} p_{25}$$
$$(k_{\text{resonance}}) \qquad k_r = \epsilon\eta_r p_{28}(1 - p_{25})$$

where p_{28} and p_{25} are the resonance escape probabilities for the fertile and fissile materials, respectively; the η's are for average thermal and resonance fissions, respectively; and the other symbols have their usual meanings.

Commonly the total k is obtained by adding k_t and k_r, but in the following it is shown that this is correct only in a very restricted sense and that a more general formulation is of the form

$$k_{\text{eff}} = \frac{k_t}{1 - k_r}$$

in those cases where $k_r < 1$, and where the average lifetime of neutrons captured at resonance energies is short compared to the thermal neutron lifetime.

This is most easily shown by adding a term for resonance fission to the usual two-group formulation:

(Fast)

$$-D_f\nabla^2\phi_f + \Sigma_f\phi_f = \epsilon\eta_t f\Sigma_t\phi_t + \epsilon\eta_r\Sigma_f p_{28}(1 - p_{25})\phi_f \qquad (1a)$$

(Thermal)

$$-D_t\nabla^2\phi_t + \Sigma_t\phi_t = p_{28}p_{25}\Sigma_f\phi_f \qquad (1b)$$

Introducing the buckling, B^2, in the usual way and solving the simultaneous equations leads to a criticality condition:

$$[1 + \tau B^2 - \epsilon\eta_r p_{28}(1 - p_{25})][1 + L^2B^2] - \epsilon\eta_t f p_{28}p_{25} = 0 \qquad (2)$$

and to terms of first order,

$$B^2 = \frac{k_t + k_r - 1}{(1 - k_r)L^2 + \tau} \qquad (3a)$$

In reactors where $k_r \ll 1$, the correction term to L^2 may be ignored, and in this sense then the k_t and k_r are simply additive. However, a more general interpretation of Eq. (3) may be obtained by solving (1a) for the ratio of the fluxes

$$\phi_f/\phi_t = \frac{k_t/p_{28}p_{25}}{(1 - k_r + \tau B^2)} \cdot \frac{\Sigma_t}{\Sigma_f} \qquad (4)$$

When leakage may be neglected, this ratio is larger by the factor $1/(1 - k_r)$ than in the corresponding case without resonance fission and suggests the interpretation of the effect as a convergent, iterative process with the multiplication at each cycle being k_r. With leakage included, it is just $k_r - \tau B^2$. This conclusion is reinforced by the observation that, for zero leakage, the critical condition can be written in the form

$$k_t/(1 - k_r) = 1$$

This suggests that a physically more meaningful expression for B^2 can be obtained by a rearrangement of Eq. (3) giving

$$B^2 = \frac{[k_t/(1 - k_r)] - 1}{L^2 + [1/(1 - k_r)]\tau} \tag{3b}$$

with increased fast leakage arising from the increased fast-to-thermal flux ratio.

If the two-group equations are written in the time-dependent form, omitting the complications arising from the delayed neutrons, then the reactor time constant, λ, can be shown to have the form

$$\lambda = \frac{1}{l_t}\left[\frac{k_t}{1 - k_r + \tau B^2} - (1 + L^2B^2)\right] \tag{5}$$

where l_t is the thermal neutron lifetime, and $l_t \gg l_f$.

It is of note that from Eq. (4) the ratio of resonance to thermal captures in the fertile material is increased by a factor of $1/(1 - k_r)$ as compared to the usual formulation.

In the above, it has been assumed throughout that the resonance captures in fertile material come at higher energies than those in the fissile material. If this is not a good model of the actual process, then p_{28} in the expression for k_r may properly be somewhat larger than in the expression for k_t.

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Prediction of Thermal-Neutron Fluxes in the Bulk Shielding Facility from Lid Tank Shielding Facility Data

In order to obtain the maximum usefulness from shielding data collected at either the Lid Tank Shielding Facility (LTSF) or the Bulk Shielding Facility (BSF), the power of the experimental source must be accurately known. Furthermore, the data must be correctly converted by geometrical transformations from the experimental source to the reactor for which the shield is being designed. One method for checking the powers quoted for the LTSF source (a disk-shaped uranium plate) and the BSF reactor, as well as a method for checking on the validity of the geometrical transformations, is to perform a calculation predicting the neutron flux in the BSF on the basis of LTSF data transformed first to a point-to-point kernel and then to the geometry of the BSF reactor. A discrepancy between the predicted and the measured fluxes would indicate either that one of the quoted powers was in error or that the geometrical transformations were not properly derived.

A calculation (*1*) that compared the fast-neutron doses was performed in 1952. Since neither the power of the Bulk Shielding Reactor (BSR) nor the power of the LTSF source plate was accurately known at that time, it was not surprising that the calculated and experimental results were not in agreement. Re-estimates of both the power of the BSR (*2*) and the power of the LTSF source plate (*3*) have since become available, however, and a second calculation has been made in which the thermal-neutron fluxes at the two facilities have been compared.

Before a conversion from LTSF data to BSF data could be made it was necessary to obtain information about the self-attenuation of neutrons inside the BSR. This was done by placing an all-aluminum mockup of one layer of fuel elements of the BSR adjacent to the source plate in the LTSF and taking thermal-neutron measurements in the water beyond the mockup. Three configurations were used: no mockup; a nine-plate mockup (one-half of a layer of elements); and an 18-plate mockup (a full layer of elements).

Since the experimental data were for a plane disk source it was necessary to convert it to a point source geometry by the standard transformation (*4*):

$$G(z) = \sum_{\nu=0}^{\infty} B(\sqrt{z^2 + \nu a^2}) \tag{1}$$

where

$$B(z') = -\frac{1}{2\pi S z'}\frac{d}{dz'} D_{pl}(z', a)$$

$z' = \sqrt{z^2 + \nu a^2}$

$G(z)$ = attenuation kernel for a point fission source at a distance z from the detector, neutrons/cm²/sec/watt

S = LTSF specific source strength, watts/cm²

$D_{pl}(z, a)$ = flux at a distance z from the disk, neutrons/cm²/sec

a = radius of disk = 35.56 cm

The source strength of the Lid Tank disk was taken to be 5.22 w ±5%, the value quoted by Otis (*3*). Since Otis calculated the source plate leakage factor for neutrons to be 0.94, the actual source strength value used was 4.91 w. The assumption was also made that the source plate power was uniform over the whole disk. Otis calculated that the source power was constant within approximately 10% from the mean.

Thermal-neutron flux attenuation kernels were calculated for the three configurations measured from Eq. (1) and then extrapolated for fuel element thicknesses up to six elements, which was the thickness of the BSR Loading 33. This extrapolation was made approximately linear on semilog graph paper. The error involved here was kept low by the fact that the largest portion of the measured flux in the BSF came from elements near the front face of the reactor core.

Equation (1) was not accurate for distances greater than about 100 cm from the source plate, however, since, in order to compute $G(z)$, it was necessary to have a knowledge of the flux, D, for fairly large distances beyond z, and such measurements were not made at points farther out than 150 cm from the source plate.

The flux data was therefore extrapolated out to about 190 cm before this calculation was performed. In order to check the validity of this extrapolation, the kernels were