Letter to the Editor

Improved Techniques of Analog and Digital Dynamic Compensation for Delayed Self-Powered Neutron Detectors

I. INTRODUCTION

This comment refers to a paper by Yusuf and Wehe.[1] After some critical remarks about this paper, an improved model of dynamic signal compensation is presented and illustrated by examples of analog and digital correction methods.

It is possible to invert a dynamic (prompt jump response) system by transforming the output equation of the state equation system to the input. Then, the overall system corresponds to a static element. Following Brockett,[2] the output equation of a non-prompt jump response system has to be differentiated until the corresponding transformation to the input will be possible. But, the differentiation aggravates the noise gain of the inverse system. Furthermore, no static transfer is possible for the overall system because in practice, the differentiation can be done only by a DT1-lag.

The physical model of the rhodium self-powered neutron detector (RSPND) has a prompt jump response. This capability is abandoned by Yusuf and Wehe. Therefore, their method of analog and digital dynamic compensation is characterized by the disadvantages named above. Now, some solutions are presented based on the original prompt jump response system.

II. STATE EQUATION SYSTEM OF A RHODIUM SELF-POWERED NEUTRON DETECTOR

Referring to Yusuf and Wehe and neglecting the insignificant terms, the state equations are

$$\frac{dN_m(t)}{dt} = S_m N_0 F(t) - l_m N_m(t) \ , \tag{1}$$

$$\frac{dN_g(t)}{dt} = S_g N_0 F(t) + l_m N_m(t) - l_g N_g(t)\, dt \ , \tag{2}$$

and

$$I(t) = K_g l_g N_g(t) + K_p(S_g + S_m)N_0 F(t) = I_d + I_p \ , \tag{3}$$

where

N_0 = atomic density of ^{103}Rh

N_g = atomic density of ^{104}Rh

N_m = atomic density of ^{104m}Rh

S_g = absorption cross section of ^{103}Rh to produce ^{104}Rh ($S_g = 139$ b)

S_m = absorption cross section of ^{103}Rh to produce ^{104m}Rh ($S_m = 11$ b)

l_g = decay constant of ^{104}Rh ($l_g = 0.0165\ \text{s}^{-1}$)

l_m = decay constant of ^{104m}Rh ($l_m = 0.002626\ \text{s}^{-1}$)

F = neutron flux

I = current

I_d = delayed component of I

I_p = prompt component of I

K_g = probability that a ^{104}Rh decay leads to a current-carrying electron

K_p = probability that a ^{103}Rh capture leads directly to a current-carrying electron.

The transfer function is

$$\frac{I(s)}{F(s)} = \frac{K_g l_g S_g N_0}{s + l_g} + \frac{K_g l_g l_m S_m N_0}{(s + l_g)(s + l_m)} + K_p(S_g + S_m)N_0 \ . \tag{4}$$

III. ANALOG METHOD

Yusuf and Wehe simplify Eq. (4) by setting $K_p = 0$, and therefore, they abandon the prompt jump response of the system. For the inversion of the model, they introduce an additional dynamic term, which at last represents the dynamics in the overall transfer function. The realization of the inverse model is done strenuously, with analog standard type electronic elements.

We get the inverse transfer function from the transfer function of Eq. (4) without abandonment of the prompt jump response:

$$\frac{F(s)}{I(s)} = \frac{\dfrac{1}{K_p(S_g + S_m)N_0}}{1 + \dfrac{K_g S_m}{K_p(S_g + S_m)}\left(\dfrac{\dfrac{S_g}{S_m} - \dfrac{l_m}{l_g - l_m}}{1 + \dfrac{s}{l_g}} + \dfrac{\dfrac{l_g}{l_g - l_m}}{1 + \dfrac{s}{l_m}}\right)} \ . \tag{5}$$

Using the prompt current component I_p of Eq. (3), it follows

$$\frac{F}{I} = \frac{1}{K_p(S_g + S_m)N_0}\,\frac{I_p}{I} \ . \tag{6}$$

Now, we introduce the parameter of the fraction of the steady-state prompt current component I_{p0} of the steady-state current I_0. With Eq. (5), Eq. (6), and the second limit theorem of Laplacian transformation, we get

$$M=\frac{I_{p0}}{I_0}=\lim_{s\to 0} s\,\frac{I_p(s)}{I(s)}=\frac{1}{1+\dfrac{K_g S_m}{K_p(S_g+S_m)}-\left(1+\dfrac{S_g}{S_m}\right)} \tag{7}$$

and

$$\frac{K_g S_m}{K_p(S_g+S_m)}=\frac{\dfrac{1}{M}-1}{1+\dfrac{S_g}{S_m}} . \tag{8}$$

For the design of an analog circuit, the representation of neutron flux F in the form of voltage U_a is necessary:

$$\frac{U_a}{I}=\frac{U_a}{F}\frac{F}{I} . \tag{9}$$

From Eq. (9) with the use of Eqs. (5) and (6) and the parameter

$$k=\frac{-U_a}{FK_g S_m N_0\left(1+\dfrac{S_g}{S_m}\right)} , \tag{10}$$

we get the transfer function

$$\frac{U_a(s)}{I(s)}=\frac{-\left(\dfrac{1}{M}-1\right)k}{1+\left(\dfrac{1}{M}-1\right)k\left[\dfrac{\dfrac{S_g}{S_m}-\dfrac{l_m}{l_g-l_m}}{k\left(1+\dfrac{S_g}{S_m}\right)}\,\dfrac{1}{1+\dfrac{s}{l_g}}+\dfrac{\dfrac{l_m}{l_g-l_m}}{k\left(1+\dfrac{S_g}{S_m}\right)}\,\dfrac{1}{1+\dfrac{s}{l_m}}\right]} . \tag{11}$$

In Fig. 1, an analog circuit is shown in which the transfer function

$$\frac{U_a(s)}{I(s)}=\frac{-R_0}{1+R_0\left(\dfrac{1}{2R_1}\,\dfrac{1}{1+\dfrac{R_1C_1}{2}s}+\dfrac{1}{2R_2}\,\dfrac{1}{1+\dfrac{R_2C_2}{2}s}\right)} \tag{12}$$

corresponds to Eq. (11). This circuit is suitable for the realization of inverse detector kinetics.

To determine the parameter k, we first estimate the static gain in Eq. (11): Using the static values U_{a0}, I_0, and the corresponding parameter M_0, it follows

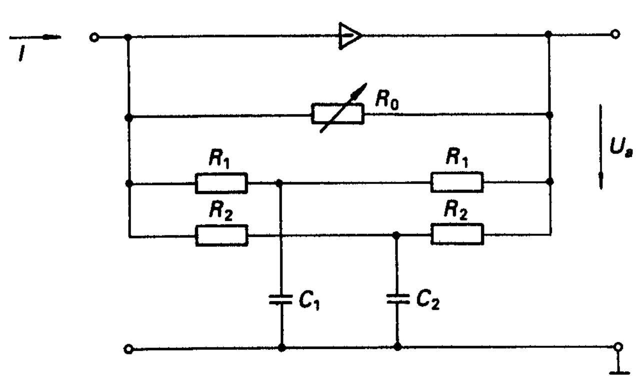

Fig. 1. Analog compensation for delayed RSPND.

$$\frac{U_{a0}}{I_0}=\lim_{s\to 0} s\,\frac{U_a(s)}{I(s)}=(1-M_0)k \tag{13}$$

and

$$k=\frac{1}{1-M_0}\,\frac{U_{a0}}{I_0} . \tag{14}$$

The parameter k can be determined by demanding a steady-state output voltage U_{a0}, corresponding to a steady-state current I_0 and a steady-state fraction M_0 of the prompt current component I_p.

After comparison of the coefficients of Eq. (11), Eq. (12), and Eq. (14), we get (just like an outline order):

$$R_0=\left(\frac{1}{M}-1\right)k , \tag{15}$$

$$R_1=\frac{\left(1+\dfrac{S_g}{S_m}\right)\left(1-\dfrac{l_m}{l_g}\right)}{\left(1+\dfrac{S_g}{S_m}\right)\left(1-\dfrac{l_m}{l_g}\right)-1}\,\frac{k}{2} , \tag{16}$$

$$R_2=\left(1+\frac{S_g}{S_m}\right)\left(1-\frac{l_m}{l_g}\right)\frac{k}{2} , \tag{17}$$

$$C_1=\frac{2}{l_g R_1} , \tag{18}$$

and

$$C_2=\frac{2}{l_m R_2} . \tag{19}$$

If we have to vary the value M, only the adjustment of resistance R_0 is necessary [Eq. (15)].

An example of the outline follows. For $I_0 = 2\times 10^{-6}$ A and $M_0 = 0.075$, we demand $U_{a0} = 1$ V. The fraction of prompt current should be adjustable in the interval $0.06 < M < 0.1$. By means of Eqs. (14) through (19), we get

$k = 0.541$ MΩ

$R_0 < 8.84$ MΩ for $M > 0.06$

$R_0 > 4.87$ MΩ for $M < 0.1$

$R_1 = 0.296$ MΩ

$R_2 = 3.10$ MΩ

$C_1 = 410$ μF

$C_2 = 246$ μF.

IV. DIGITAL METHOD

As mentioned, Yusuf and Wehe simplify Eqs. (1), (2), and (3) by setting $K_p = 0$. Their equation of inverse detector kinetics thus contains a term $dI(t)/dt$, by which the noise transfer of the inverse system could be injured considerably. Yusuf and Wehe do not give information about the numerical realization of the inverse model. But, this is important for the judgment of computational speed and accuracy with regard to real-time processing.

We transform the transfer function Eq. (4) without abandonment by reducing to partial fractions in

$$\frac{F(s)}{I(s)} = \frac{1}{K_p(S_g + S_m)N_0}\left(1 + \frac{A}{s+a} + \frac{B}{s+b}\right) , \quad (20)$$

where by means of Eq. (8),

$$a;b = \frac{1}{2}\left\{l_m + \left[1 + \frac{K_g S_g}{K_p(S_g+S_m)}\right]l_g\right\} \pm \left(\left\{\frac{1}{4}\left\{l_m + \left[1 + \frac{K_g S_g}{K_p(S_g+S_m)}\right]l_g\right\}\right)^2 - \left[1 + \left(1 + \frac{S_g}{S_m}\right)\frac{K_g S_m}{K_p(S_g+S_m)}\right]l_g l_m\right)^{0.5}$$

$$= \frac{1}{2}\left(l_m + l_g \frac{1 + \frac{S_g}{S_m}\frac{1}{M}}{1 + \frac{S_g}{S_m}}\right) \pm \left[\frac{1}{4}\left(l_m + l_g \frac{1 + \frac{S_g}{S_m}\frac{1}{M}}{1 + \frac{S_g}{S_m}}\right)^2 - l_g l_m \frac{1}{M}\right]^{0.5} , \quad (21)$$

$$A = l_g l_m \frac{K_g S_m}{K_p(S_g+S_m)} \frac{a\frac{S_g}{l_m S_m} - 1 - \frac{S_g}{S_m}}{b-a}$$

$$= l_g l_m\left(\frac{1}{M} - 1\right)\left(\frac{a\frac{l_g S_g}{l_m S_m}}{1 + \frac{S_g}{S_m}} - 1\right)\frac{1}{b-a} , \quad (22)$$

and

$$B = -l_g l_m \frac{K_g S_m}{K_p(S_g+S_m)} - \frac{b\frac{S_g}{l_m S_m} - 1 - \frac{S_g}{S_m}}{b-a}$$

$$= -l_g l_m\left(\frac{1}{M} - 1\right)\left(\frac{b\frac{S_g}{l_m S_m}}{1 + \frac{S_g}{S_m}} - 1\right)\frac{1}{b-a} . \quad (23)$$

Using the parameter

$$k_d = \frac{1}{K_g S_m N_0\left(1 + \frac{S_g}{S_m}\right)} , \quad (24)$$

it follows that

$$\frac{1}{K_p(S_g+S_m)N_0} = \left(\frac{1}{M} - 1\right)k_d . \quad (25)$$

On the conditions of equidistant sampling at times $t = nT_a$ ($n = 0,1,2,\ldots$; T_a = sampling period) and rectangular approximation of current $I(t)$, and by means of the so-called Z transformation, we get the result of pulse transfer function

$$Z\left[\frac{1 - e^{-sT_a}}{s}\left(1 + \frac{A}{s+a} + \frac{B}{s+b}\right)\right] = 1 + \frac{A}{a}\frac{1 - z_a}{z - z_a} + \frac{B}{b}\frac{1 - z_b}{z - z_b} , \quad (26)$$

where

$$z_a = e^{-aT_a}$$

and

$$z_b = e^{-bT_a} .$$

Now, we introduce the state variables x_a and x_b in the second and the third terms in Eq. (26). Using Eq. (26), we obtain the recursive algorithm

$$x_a(n+1) = z_a x_a(n) + (1 - z_a)\frac{A}{a}I(n) , \quad (27)$$

$$x_b(n+1) = z_b x_b(n) + (1 - z_b)\frac{B}{b}I(n) , \quad (28)$$

and

$$F(n) = \left(\frac{1}{M} - 1\right)k_d[I(n) + x_a(n) + x_b(n)] , \quad (29)$$

and for the steady state

$$x_a(0) = \frac{A}{a}I(0)$$

and

$$x_b(0) = \frac{B}{b}I(0) .$$

An example of the outline follows. For $M = 0.075$ and $T_a = 0.1$ s, we get the constants

$a = 0.0028198\ s^{-1}$
$b = 0.20488\ s^{-1}$
$A = -0.000013116\ s^{-1}$
$B = -0.18856\ s^{-1}$
$z_a = 0.99972$
$z_b = 0.97972$

and the recursive algorithm is

$$x_a(n+1) = 0.99972x_a(n) - 1.311 \times 10^{-6}I(n) ,$$
$$x_b(n+1) = 0.97972x_b(n) - 0.018664I(n) ,$$

and

$$F(n) = 12.33k_d[I(n) + x_a(n) + x_b(n)] ,$$

with the initial values

$$x_a(0) = -0.004651I(0)$$

and

$$x_b(0) = -0.920348I(0) .$$

In practice, the parameter k_d can be determined by Eq. (29) by scaling the measured steady-state current $I(0)$ to the known steady-state neutron flux $F(0)$.

V. SUMMARY AND DISCUSSION

The analog circuit presented here contains only one active electronic element, which is responsible for the compensation of detector dynamics, the signal amplification and the current-to-voltage transformation. This circuit can be considered as the minimum of effort. By means of this circuit, it is possible to measure the time-dependent neutron flux behavior without delay and with high accuracy. The steady-state prompt fraction of the RSPND current is adjustable with the help of only one resistance.

The given discrete algorithm includes no differentiation of detector current and thus has a good noise gain. These improved analog and digital dynamic compensation methods of RSPND were developed and used in German and Hungarian nuclear power plants with pressurized water reactors of Soviet VVER type.[3,4] By means of many rhodium detectors and the named correction methods, the time- and space-dependent neutron flux behavior during power changes or reactivity perturbations was followed to estimate important reactivity coefficients like differential control rod worths or power coefficient. Furthermore, both the developed compensation principles and the reactor-dynamic perturbation method allow the estimation of the very important detector value of the steady-state prompt fraction of RSPND current by an experimental process analysis.

Dietrich Hoppe
Rainer Maletti

Central Institute of Nuclear Research Rossendorf
Department of Reactor Physics
Box 19
D-8051 Dresden, Federal Republic of Germany

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